Differential Test for Series of Positive Terms, New Tree-Field Representations in Graph Theory and New Number Field, Extension of Dirac Extraction, and Their Applications

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Abstract: First, a new differential test for series of positive terms is proved. Let *f(x)* be a positive continuous function corresponded to a series of positive terms $\sum_{k=1}^{\infty} f(k)$, and *g(x)* is a derivative of reciprocal of *f(x)*, i.e., $\frac{d}{dx}[\frac{1}{f(x)}]=g(x)$. Then, if $fgx \geq 1+\alpha(\alpha>1)$ for enough large x, the series converges; if $fgx \leq 1$, the series diverges. The rest may make the limit form, and is universal and complete. Next, the tree in the graph theory is extended to a new tree-field representation. It includes two parts: tree and field. A field is a set of legion small trees. They can transform each other between tree and field. This is a unification of simplicity (tree) and complexity (field), and may be applied to various complex systems on science, politics, economy, philosophy and so on. Further, it may be extended to the whole graph theory G=(V,E,F), here F is a set of small graphs. Third, based on a brief review on developments of number system, a new developed pattern is proposed. The quaternion is extended to a matrix form *aI+bC+cB+dA*, in which the unit matrix I and three special matrices C,B,A correspond to number 1 and three units of imaginary number i,j,k, respectively. They form usually a ring. But some fields may be composed of some special 2-rank, even n-rank matrices, for example, three matrices *aI+bC*, *aI+cB*, *aI+dA* and so on. It is a new type of hypercomplex number fields. The physical applications and possible meaning of the new number system is researched. Finally, the Dirac extraction is extended to any terms $A^2=B^2+C^2+D^2+...$ whose extraction should be $A=\alpha B+\beta C+\gamma D+...$ and $\alpha^2=\beta^2=\gamma^2=...=1$, etc. Moreover, the general complexity is also discussed.

Key words: series of positive terms; convergence and divergence; differential; infinite integral; graph theory; tree; number system; matrix; complex number; ring; field; extraction; application; complexity

1.New differential test for series of positive terms and its completeness

In mathematics some basic problems are still worth research. The most well-known one is the nonstandard analysis (NSA) [1]. The convergence or divergence for a series of positive terms is elementary in calculus, and has been many tests. Besides the basic comparison test, there are the D'Alembert ratio rest, the integral test [2], the Cauchy root test, the Raabe test, the Kummer test, the Abel-Dini test and so on [3,4]. But, the applicable regions of these tests are not usually the same, and are generally finite. Using a similar method with the integral test, we propose a new differential test for series of positive terms, and discuss some examples.

The differential test. Let $\sum_{k=1}^{\infty} f(k)$ be a series of positive terms, *f(x)* is a corresponding positive continuous function, and *g(x)* is a derivative of reciprocal of *f(x)*, i.e., $\frac{d}{dx}[\frac{1}{f(x)}]=g(x)$. Then, if $fgx \geq 1+\alpha(\alpha>0)$ for enough large x, the series converges; if $fgx \leq 1$ the series diverges.

Proof: Since $g(x)=\frac{d}{dx}[\frac{1}{f(x)}]=-\frac{d}{dx}[\ln f(x)]\frac{1}{f(x)}$, if $fg=-\frac{d}{dx}[\ln f(x)] \geq \frac{1+\alpha}{x}$, then the integral from 1 to x is obtained $-\ln\frac{f(x)}{f(1)} \geq (1+\alpha)\ln x$, $\therefore f(x) \leq \frac{f(1)}{x^{1+\alpha}}$, the series $\left\{\frac{1}{k^{1+\alpha}}\right\}$ and {*f(k)*} converge; if $fg=-\frac{d}{dx}[\ln f(x)] \leq \frac{1}{x}$, the integral is obtained $\ln\frac{f(1)}{f(x)} \leq \ln x$, $\therefore f(x) \geq \frac{f(1)}{x}$, the series $\left\{\frac{1}{k}\right\}$ and {*f(k)*} diverge.

In calculus the differentiation is simple, and can apply to various composite functions of any elementary functions, therefore, the test is also very simple, and the applicable region is wide. Moreover, a calculating result for any series must be $fgx \geq 1+\alpha(\alpha>0)$ or $fgx \leq 1$, so the rest should be universal and complete [5].

If f is a discrete function a_n, the difference is substituted for differentiation $g=\frac{a_{n+1}^{-1}-a_n^{-1}}{1}=\frac{a_n-a_{n+1}}{a_n a_{n+1}}$, $\lim_{n\to\infty} nfg=\lim_{n\to\infty} n(\frac{a_n}{a_{n+1}}-1)=c$, i.e., the Raabe test.

In many cases, the test may make the limit form, i.e., *fgx*→*c* as $x\to\infty$. Then the series converges if c>1 and diverges if c<1. Therefore, it may combine the L'Hopital's rule. Of course, it cannot test for c=1, since an infinitesimal α is neglected in the limit form. But, we use a general differential test, its results can but be $fgx=1+\alpha$, in which α >0 or $\alpha \leq 0$. Such when we combine the nonstandard analysis in which α is hyperreal number of infinitesimal [1], the differential test must be completeness.

Example 1. The series $f(n)=a^n b^{n-1}+a^n b^n$ cannot apply the ratio test. Let $f(x)=a^x b^{x-1}+a^x b^x$, $g(x)=-\frac{\ln(ab)}{a^x b^{x-1}(1+b)}$, $fgx=-x\ln(ab), \therefore$ the series converges if

$ab<1$, and diverges if $ab \geq 1$.

Example 2. The series $f(n)=\frac{1}{n!}(na)^n,(a>0)$ must apply the Raabe test. Let $f(x)=\frac{1}{\Gamma(x+1)}(xa)^x$, $g(x)=(xa)^{-x}[\Gamma'(x+1)-(1+\ln ax)\Gamma(x+1)]$. When x is very large number, $\Gamma'(x+1)\approx[(\frac{x}{e})^x\sqrt{2\pi x}]'=\Gamma(x+1)(\frac{1}{2x}+\ln x)$, $fgx=\frac{1}{2}-x\ln(ae),\therefore$ $\lim_{x\to\infty} fgx=\infty$ for $a<1/e$, the series converges; fgx $\leq 1/2$ for $a \geq 1/e$ the series diverges.

Example 3. The series $f(n)=\frac{1}{n^p}\sin\frac{\pi}{n}$, let $f(x)=\frac{1}{x^p}\sin\frac{\pi}{x}$, $g(x)=\frac{x^{p-1}}{\sin(\pi/x)}[p+\frac{\pi}{x}\text{ctg}\frac{\pi}{x}]$, $fgx=p+\frac{\pi}{x}ctg\frac{\pi}{x}$, $\lim_{x\to\infty}\frac{\pi}{x}ctg\frac{\pi}{x}=\lim_{x\to\infty}\cos^2\frac{\pi}{x}$ =1, fgx=1+p, the series converges for $p>0$, and diverges for $p\leq 0$.

Further, the differentiation test may be applied to test for the general series of functional terms, and test for the infinite integral whose limits of integral from positive to infinite. The convergence or divergence of the integral is determined by the differential of the integrand.

Example 4. The integral $\int_1^\infty \frac{dx}{x^p+x^q}$, $f(x)=\frac{1}{x^p+x^q}$, $g(x)=px^{p-1}+qx^{q-1}$, $fgx=\frac{px^p+qx^q}{x^p+x^q}$. Using the L'Hopital's rule, fgx→c=p for $p>q$ or c=q for $p<q$, so max $(p,q)>1$, the infinite integral converges.

2.New tree-field representation in graph theory and its applications

A known important representation is tree in the graph theory [6]. We extend tree, and propose a new tree-field representation. It includes two parts: tree (trunk) and field. The Figure 1 of the tree-field is:

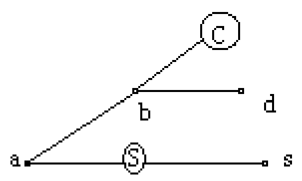

Fig.1. Tree-field representation

Here the fields (C and S, etc.) are some sets of much small trees, which are also trees under microscope. For a certainty condition the few small trees in the field can become a usual tree as trunk. These may be the directed graph or undirected graph.

In Fig.1 the vertexes b, d, s and the fields C, S possess the same macroscopic property, and fields are only minute detail on vertex. In the tree-field representation there is a relation (for vertex n, edge m and field s):

$$m = n + s - 1. \quad (1)$$

Further, the field may have different weights and levels, which are blown up and the tree-field appear. This can mathematically combine the nonstandard analysis [1], and may apply the fractal. It may be extended to the graph theory, i.e., G=(V,E,F), in which F is field and a set of the small graph, and represents region, grove and forests, etc. This is not the supergraph [7], but is a bit similarity with the dense graphs [7].

For the developments of science, tree represents principal science, and field corresponds to secondary science, which includes pre-science and potential science and so on. Tree is the paradigm in Kuhn's science development. For synergetics tree is an order parameter and field is the general parameter. Tree and field represent respectively main space-time and other space-time, main aspect and other aspect, main movement and other movement, main form and other form, etc. The philosophical meaning is that tree represents main research direction: human nature and whole order of cosmology [8], both corresponds to the philosophy of the mind and the philosophy of the Qi in Chinese traditional philosophy, their unification is namely Neo-Confucianism; field represents much other philosophical regions. In epistemology, tree is a few reality, and field is multicoloured appearance [8]. In these regions both can transform each other. It is a unified of simplicity (tree) and complexity (field).

Usual science systems, for example, biology, have the tree-field representations of many levels, in which tree represents main sequence and field corresponds to numerous variation. In hypercycle [9] some representations have already similar with the vertex-field of the graph theory. For instance, an extensive evolution principle of hypercycle (Fig. 2)[9,10], here three E are vertex, and three I correspond to field, which may be still various hypercycles and other structures.

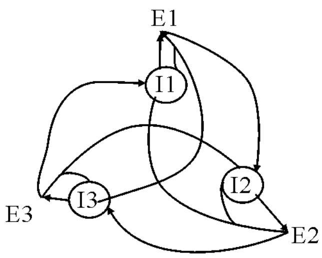

Fig. 2. Extensive evolution principle of hypercycle

In economy the tree represents the main aspect of the economic growth for a time, and the field represents other economic aspects, but which will produce probably new direction of economic growth [11,12]. For the political economics as a multiply connected topological economics [12,13], the politics is often first, and corresponds to tree, and economy is attached, and corresponds to field. In a word, the tree-field representation may be applied to many aspects, in particular, for analysis of various complex systems.

3.Various number systems

In mathematics first base is number [14]. Basic fields are the field of rational numbers Q, the field of real numbers R and the field of complex numbers C in the number system [15]. Further developments rise and fall. In 1843 W.R.Hamilton obtained a quaternion $a+bi+cj+dk$, here $i^2 = j^2 = k^2 = -1$. But, the quaternion is a ring. He and Maxwell, Heaviside, et al., discussed various applications of the quaternion. Then Graves and Cayley derived biquaternions. Grassmann researched another hypercomplex. In 1873 W.K.Clifford obtained the Clifford number. It is yet other hypercomplex, or the quasi-quaternion $q+\omega Q$, here q and Q are real quaternion, and $\omega^2=1$. In the Clifford algebra there have elements 1, $e_1,e_2,...e_{n-1}$, here $e_i^2=-1$, $e_ie_j=-e_je_i$. In 1889 A.Hurwitz proved that real number, complex number, real quaternion and Clifford quasi-quaternion are only a linear algebra satisfied the multiplication axiom [16]. Then there are the field of algebraic numbers, the field of rational functions, the field of trigonometric functions, the field of meromorphic functions and Hensel p-adic fields and so on [14,15].

In algebra there is Frobenius Theorem: The only division algebras of finite rank over the real number field are itself, the complex number field, and the quaternion algebra [15]. But, in the nonstandard analysis (NSA) there is already new hyperreal number: infinitesimal and infinite [1]. Now the extension of field is an important part of algebra [17]. Here we research that the complex number system is developed to a method by the representation of matrix, and derive a type of the hypercomplex fields. It is a new extended pattern on the number system.

Using a matrix representation of quaternion, the complex number system is represented by matrix, here 1→I=$\begin{pmatrix}1&0\\0&1\end{pmatrix}$ is unit matrix, and i→C=$\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ and $C^2=-I$. Assume that a new complex positive number j, here j^2=1. Then, let j→B=$\begin{pmatrix}0&1\\1&0\end{pmatrix}$, B^2=I, correspond to an inversion operator. k=ij→A=$\begin{pmatrix}-1&0\\0&1\end{pmatrix}$=CB, here $A^2=I$ and ji=－k→－A. A set (1,i,j,k) is another quaternion, and relates the Clifford fourfold quaternion [18,19]. This corresponds to that a field is extended to a ring.

Further, the matrix representation of this number system may extend to higher rank matrix and high dimensional space, and both may be combined each other.

(1).We extend to higher unit matrix I, and a main diagonal matrix becomes a secondary diagonal matrix, i.e.,

$$B=\begin{pmatrix} & & 1\\ & 1 & \\ 1 & & \end{pmatrix} \text{ and so on, to } \begin{pmatrix} & & & 1\\ & & 1 & \\ & ... & & \\ 1 & & & \end{pmatrix}. \qquad (2)$$

Moreover, some main diagonal matrices are introduced:

$$A=\begin{pmatrix}-1 & & \\ & -1 & \\ & & 1\end{pmatrix} \text{ or } \begin{pmatrix}-1 & & \\ & 1 & \\ & & 1\end{pmatrix} \text{ and so on, to } \begin{pmatrix}-1 & & & \\ & -1 & & \\ & & ... & \\ & & & 1\end{pmatrix}, \qquad (3)$$

and these matrices become the secondary diagonal matrices, i.e.,

$$C=\begin{pmatrix} & & -1\\ & -1 & \\ 1 & & \end{pmatrix} \text{ or } \begin{pmatrix} & & -1\\ & 1 & \\ 1 & & \end{pmatrix} \text{ and so on, to } \begin{pmatrix} & & & -1\\ & & -1 & \\ & \cdots & & \\ 1 & & & \end{pmatrix}. \quad (4)$$

Here B^2=I, A^2=I, only the changes of $\begin{pmatrix} & & -1\\ & -1 & \\ 1 & & \end{pmatrix}^2=\begin{pmatrix} -1 & & \\ & 1 & \\ & & -1\end{pmatrix}=\begin{pmatrix} & & -1\\ & 1 & \\ 1 & & \end{pmatrix}^2$ and

$$\begin{pmatrix} & & & -1\\ & & -1 & \\ & -1 & & \\ 1 & & & \end{pmatrix}^2=\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1\end{pmatrix}$$ and so on, are the biggest.

(2).Bothe combines each other

$$I+C=\begin{pmatrix} 1 & -1\\ 1 & 1\end{pmatrix},\ I+B=\begin{pmatrix} 1 & 1\\ 1 & 1\end{pmatrix},\ I+A=\begin{pmatrix} 0 & 0\\ 0 & 2\end{pmatrix}=2I_{22} \text{ and } C+B=\begin{pmatrix} 0 & 0\\ 2 & 0\end{pmatrix}=2I_{21}. \quad (5)$$

It is analogue with the elementary matrix I_{ij}, in which only these terms on i and j are 1, other elements are 0.

$$(I+C)(I+C)=2C,\ (I+B)(I+B)=2(I+B),\ (I+A)(I+A)=2(I+A). \quad (6)$$

The after two forms are DD=2D→ $D^n=2^{n-1}D$. And $(I+C)^3=2(C-I)$, $(I+C)^4=-4I$ and so on. In the quantum field theory [20] the creation operator of particle number is

$$a^+=\begin{pmatrix} 0 & 0\\ 1 & 0\end{pmatrix}=\frac{C+B}{2}, \quad (7)$$

the annihilation operator is

$$a=\begin{pmatrix} 0 & 1\\ 0 & 0\end{pmatrix}=\frac{B-C}{2}, \quad (8)$$

and the particle number operator is

$$N=a^+a=\begin{pmatrix} 0 & 0\\ 0 & 1\end{pmatrix}=\frac{I+A}{2},\quad aa^+=\begin{pmatrix} 1 & 0\\ 0 & 0\end{pmatrix}=\frac{I-A}{2}. \quad (9)$$

Eqs.(7), (8) and (9) correspond to four numbers. Moreover,

$$a^+a+aa^+=\begin{pmatrix} 1 & 0\\ 0 & 1\end{pmatrix}=I,\quad a^++a=\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}=B. \quad (10)$$

Then we may develop to various quaternion and n-element quantity, for example, I+A(I+I+I…), I+B(I+I+I+…), I+C(I+I+I+…), A+I(I+A+B+C+…), etc.

(3). A general main diagonal matrix is

$$A=\begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \dots & \\ & & & a_n \end{pmatrix}, \qquad (11)$$

and a secondary diagonal matrix is

$$B=\begin{pmatrix} & & & b_1 \\ & & b_2 & \\ & \dots & & \\ b_n & & & \end{pmatrix}. \qquad (12)$$

Their products are still main diagonal matrices or secondary diagonal matrices:

$$AA'=\begin{pmatrix} a_1a'_1 & & & \\ & a_2a'_2 & & \\ & & \dots & \\ & & & a_na'_n \end{pmatrix}, \ AB=\begin{pmatrix} & & & a_1b_1 \\ & & a_2b_2 & \\ & \dots & & \\ a_nb_n & & & \end{pmatrix},$$

$$BA=\begin{pmatrix} & & & b_1a_n \\ & & b_2a_{n-1} & \\ & \dots & & \\ b_na_1 & & & \end{pmatrix}, \ BB'=\begin{pmatrix} b_1b'_n & & & \\ & b_2b'_{n-1} & & \\ & & \dots & \\ & & & b_nb'_1 \end{pmatrix},$$

$$B'B=\begin{pmatrix} b_nb'_1 & & & \\ & b_{n-1}b'_2 & & \\ & & \dots & \\ & & & b_1b'_n \end{pmatrix}, \text{ (here } a_i, b_i \in C\text{).} \qquad (13)$$

4.Field composed by matrices

The matrix $\{aI+bC+cB+dA \mid a,b,c,d \in C\}$ forms a ring for addition and multiplication.

Here

CB=A, BC=－A; AC=B, CA=－B; AB=C, BA=－C. (14)

The multiplication of matrix is non-commutation, and is anticommutation. Since A, B and C have the inverse-elements, the division for matrix is defined by the inverse matrix. Such it is a division ring of real quaternions. But, three matrices $aI+bC$, $aI+cB$ and $aI+dA$ all have inverse-elements, and obey the commutative law and the associative law of multiplication, and construct three similar commutative quotient ring——field. An example is:

$$aI+bC=\begin{pmatrix} a+a_1i & -b-b_1i \\ b+b_1i & a+a_1i \end{pmatrix}. \qquad (15)$$

In usual case the matrix does not obey the commutative law of multiplication, but the main diagonal matrices obey the commutative law. For the n-rank matrices (13), when $a_1=a_n$,

$a_2 = a_{n-1}$,…i.e., AB=BA, and $b_1b'_n = b_nb'_1$, $b_2b'_{n-1} = b_{n-1}b'_2$, …i.e., $BB' = B'B$, it obeys the commutative law. In these cases a new number field may be constructed.

In more general case, for example, some special 2-rank matrices are

$$J=\begin{pmatrix} bh+d & b \\ kb & d \end{pmatrix}, \qquad (16)$$

in where h and k are constant. It obeys the commutative law of multiplication. Therefore, $\{aI + cJ \mid a,c \in C\}$ constructs the field. These matrices include: 1.A supertriangle matrix (for k=0), which may become a main diagonal matrix. 2.J=$b\begin{pmatrix} h & 1 \\ k & 0 \end{pmatrix}$ (for d=0). 3. J=$\begin{pmatrix} d & b \\ kb & d \end{pmatrix}$ (for h=0). Further, this method may be extended to 3-rank, n-rank special matrices. These matrices have unit matrices I, zero element, negative element and inverse element, and addition, etc., and they obey yet commutative law of multiplication, so they may construct a number field. Any rank diagonal matrices all obey the commutative law of multiplication, and may construct the field.

Usual matrix obeys the algorithm of ring. This pattern may construct a big type of general ring by matrix representation, for instance, algebra of Pauli ring and Dirac ring [18]. Moreover, they may construct the field by some special matrices under the certain conditions. For example, the 3-rank matrices:

$$I=\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix},\ D_1=\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix},\ D_2=\begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix},\ D_3=\begin{pmatrix} & & 1 \\ & -1 & \\ 1 & & \end{pmatrix}. \qquad (17)$$

Because $D_1D_2 = D_2D_1 = D_3$, $D_1D_3 = D_3D_1 = D_2$ and $D_2D_3 = D_3D_2 = D_1$, the matrices of quaternion form $aI + bD_1 + cD_2 + dD_3$, and in which three dual-parts $aI + bD_1$, $aI + cD_2$ and $aI + dD_3$ all construct the general fields. Simultaneously,

$$I=\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix},\ D_2=\begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix},\ D_4=\begin{pmatrix} & & 1 \\ & 1 & \\ -1 & & \end{pmatrix},\ D_5=\begin{pmatrix} & & 1 \\ & -1 & \\ -1 & & \end{pmatrix}. \qquad (18)$$

Because the products of D_2, D_4, D_5 all obey the commutative law of multiplication, a new hypercomplex number

$$aI + bD_2 + cD_4 + dD_5, \qquad (19)$$

and in which some dual-parts construct yet the fields. According to the same composing rules, a matrix of quaternion form is composed of these 4-rank matrices:

I= $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}, G_1 = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}, G_2 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}, G_3 = \begin{pmatrix} & & & -1 \\ & & 1 & \\ & 1 & & \\ -1 & & & \end{pmatrix}$,

which construct still a field. It is namely that a field is constructed by the matrix as follows:

$$aI + bG_1 + cG_2 + dG_3 = \begin{bmatrix} a-b & & & c-d \\ & a+b & c+d & \\ & c+d & a+b & \\ c-d & & & a-b \end{bmatrix}. \qquad (20)$$

For more general cases the eight matrices of 5-rank I and G_i (i=1,2,3,4,5,6,7) are composed of the main diagonal elements, which are respectively (1,1,1,1,1), (-1,1,1,1,-1), (-1,-1,1.-1,-1), (1,-1,1,-1,1) and corresponding secondary diagonal elements. This form

$$a_1 I + a_2 G_1 + a_3 G_2 + a_4 G_3 + a_5 G_4 + a_6 G_5 + a_7 G_6 + a_8 G_7, \text{ (here } a_i \in C), \qquad (21)$$

constructs yet a field. The eight matrices of 6-rank are composed of the main diagonal elements, which are respectively (1,1,1,1,1,1), (-1,1,1,1,1,-1), (-1,-1,1,1,-1,-1), (1,-1,1,1,-1,1) and corresponding secondary diagonal elements. It constructs still a field.

The twelve matrices of 7-rank are composed of the main diagonal elements, which are respectively (1,1,1,1,1,1,1), (-1,1,1,1,1,1,-1), (-1,-1,1,1,1,-1,-1), (-1,-1,-1,1,-1,-1,-1), (1,-1,-1,1,-1,-1,1), (1,-1,1,-1,1,-1,1) and corresponding secondary diagonal elements. It constructs a field. The twelve matrices of 8-rank are composed of the main diagonal elements, which are respectively (1,1,1,1,1,1,1,1), (-1,1,1,1,1,1,1,-1), (-1,-1,1,1,1,1,-1,-1), (-1,-1,-1,1,1,-1,-1,-1), (1,-1,-1,1,1,-1,-1,1), (1,-1,1,-1,-1,1,-1,1) and corresponding secondary diagonal elements. It constructs yet a field and so on. These fields are composed of even elements. In a word, the number system extends to matrix, which may construct very abundant various fields.

5.Physical applications and possible meaning of new number system

The multiplication of quaternion is non-commutative, which may describe the composition of rotation on the rigid body. In relativity if the four dimensional space-time corresponds to a quaternion, it should be that three imaginary numbers correspond to three dimensional space, and a real number corresponds to one dimensional time. Assume that a four dimensional space-time is:

$$(Ix)^2 + (By)^2 + (Cz)^2 + (Act)^2. \qquad (22)$$

This is a new representation of four dimensional space-time. A similar higher rank matrix corresponds to a metric matrix $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$ of four dimensional space-time of special relativity. It becomes a light-cone for two dimensional space-time, and four dimensional geometry corresponds to a hyperboloid of two sheets. The metric matrix of general relativity corresponds to a general 4-rank matrix, and hyper-real number. Segel discussed the infinite and the infinitesimal in models for natural phenomena [21].

In quantum mechanics matrix is a basic tool, and q-number is non-commutative. They correspond to operator and ring. Therefore, the hypercomplex system (1,i,j,k) relates three dimensional Pauli matrices in quantum mechanics [22], i.e.,

$$\sigma_x = B, \quad \sigma_y = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = iC, \quad \sigma_z = -A. \qquad (23)$$

They and I construct four dimensional space-time. Further, this relates the Dirac matrices and extensions, in which G_3 is namely γ_2 in Dirac matrices. Any products of Pauli and Dirac matrices all are closed. 1 and Pauli matrices are the basic elements of a ring, which forms the Pauli algebra. Dirac matrices construct another ring, and correspond to another algebra and Clifford number [18]. 1i=i1 is commutative, and corresponds to boson; ij= − ji is anticommutative, and corresponds to fermion. The complex number system relates the supersymmetry and Graded Lie Algebras (GLA), in which the commutation rules for the (D+d) dimensional GLA [23] are:

$$[A_m, A_n] = A_m A_n - j^2 A_n A_m = f_{mn}^{l} A_l. \qquad (24)$$

Even elements and Lie Algebra corresponds to boson and integral spin.

$$\{Q_\alpha, Q_\beta\} = Q_\alpha Q_\beta - i^2 Q_\beta Q_\alpha = F_{\alpha\beta}^{m} A_m. \qquad (25)$$

Odd elements and GLA corresponds to fermion and half-integral spin. The unification of A and Q corresponds to unification of i and j. Moreover,

$$[A_m, Q_\alpha] = A_m Q_\alpha - k^2 Q_\alpha A_m = S_{m\alpha}^{\beta} Q_\beta. \qquad (26)$$

The above development of complex number system will redound to various unified representations between Lie Algebra and GLA, between commutation and anticommutation relations, between boson and fermion, and between Bose-Einstein statistics and Fermi-Dirac statistics. This relates the supersymmetry, and a unified statistics and possible violation of Pauli exclusion principle [24-28].

In special relativity the Lorentz transformation may be represented as a matrix form. The supersymmetric change between bosons and fermions may apply matrix. Combining both, we proposed a mathematical physical law: Bosons correspond to real number, and fermions correspond to imaginary number. Such bosons and fermions consist of even and odd fermions, respectively, which just corresponds to even and odd imaginary numbers are real and imaginary number [29,30]. Combining the development of complex number, we developed the complex number from the fractal dimension, and researched its applications in mathematics and physics, and the fractal space-time theory [31,32]. From this it may be constructed that the higher dimensional, fractal, complex and hypercomplex space-time theory covers all [31-33].

In a word, 1).The number extends to a matrix representation, i.e., number 1 and imaginary number units (i,j,k) correspond to a unit-matrix I and the special matrices (C,B,A). 2).The matrix *aI+bC+cB+dA* constructs a ring, and corresponds to the forms of quaternion. 3).Three matrices *aI+bC*, *aI+cB* and *aI+dA* all are fields. 4).We may extend some special 2-rank, even higher-rank matrices, and construct some fields. 5).This new developed pattern of number system is possibly some physical meaning. We predict that if the field can be extended, this will be able to apply to many regions.

6.Extension of Dirac extraction and general complexity

Usual extraction all falls across the negative number. The operator extraction corresponds to the quantization of the electromagnetic field and other fields. Dirac proposed a method [34]

$$E^2 = c^2 p^2 + m^2 c^4, \qquad (27)$$

whose extraction is

$$E = \alpha cp + \beta mc^2. \qquad (28)$$

This obtains the famous Dirac equations in quantum mechanics. We propose this method may be generally applied: For any square:

$$A^2 = B^2 + C^2, \qquad (29)$$

whose extraction should be:

$$A = \alpha B + \beta C. \qquad (30)$$

They include various four-vector. Further, it may extend to arbitrary terms:

$$A^2 = B^2 + C^2 + D^2 + ... \qquad (31)$$

whose extraction may be:

$$A = \alpha B + \beta C + \gamma D + ... \qquad (32)$$

Here $\alpha^2 = \beta^2 = \gamma^2 = ... = 1,$ (33)

and $\alpha\beta = \beta\gamma = \gamma\alpha = ... = 0.$ (34)

Mathematically, it is similar with Euler substitution in integral:

$$\sqrt{ax^2 + bx + c} = \sqrt{a}x \pm t, xt \pm \sqrt{c}, t(x - \alpha). \qquad (35)$$

Further, for arbitrary n-power equation:

$$A^n = B^n + C^n + D^n + ... \qquad (36)$$

whose extraction of n-power may be:

$$A = \alpha_0 B + \beta_0 C + \gamma_0 D + ... \qquad (37)$$

Here $\alpha_0^n = \beta_0^n = \gamma_0^n = ... = 1,$ (38)

$$\alpha_0\beta_0 = \beta_0\gamma_0 = \gamma_0\alpha_0 = ... = 0, \quad \alpha_0^m = \beta_0^{m'} = \gamma_0^{m''} = ... = 0. \qquad (39)$$

and n>(m,m',m'')>0. Various extensions may introduce various matrix. They may be some very funny matrix, even may obtain new algebra [35].

From Leibniz, C.S.Peirce, Whitehead to J.T.Bonner [36] and J.H.Holland [37], they all researched complexity, even proposed the hypercomplexity and the natural dynamics of higher complex systems from disorder to order, from chaos to structure [38], and so on.

The systems in chemistry and biology are based on the more simple physics, and produce the structural complexity or the compositioal complexity, and form the functional complexity [39]. The social systems are more complex [40]. We proposed the nonlinear whole biology and its basic laws [41,42], which is also a mathematical method described biological complexity. The tree-field representation proposed in this paper may provide a new mathematical method described various complexities.

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