International Journal of Sciences
(ISSN 2305-3925)
http://www.ijSciences.com

Research Article
Volume 2, Issue Apr 2013

Atomic, Ionic and Bohr Radii Linked via the Golden Ratio for Elements of Groups 1 - 8 including Lanthanides and Actinides

Raji Heyrovska[1]✎

[1]Na Stahlavce 6, 16000 Praha 6, Czech Republic

Abstract: In an earlier article, atomic radii of elements of Groups 1A - 8A were shown to be proportional to their Bohr radii obtained from the first ionization potentials and the constants of proportionality were shown to be simple functions of the Golden ratio. Here it is shown that the above finding holds also for elements of Groups 1B - 8B including Lanthanides and Actinides. All the relevant data are presented in Tables and the correlations are shown in graphs. Revised data for some of the elements of Groups 1A - 8A, have also been included.

Keywords: Bond lengths, Covalent radii, Ionic radii, Bohr radii, Golden ratio, Group 1- 8 elements, Lanthanides, Actinides, Radii of electrons, Nuclear radii

1. Introduction

1.1. Golden sections of the Bohr radius from first ionization potentials

The role of the Golden ratio in molecular structures has come into recent prominence[1-3]. While pondering on the relation between electrochemical redox potentials and gaseous ionization potentials, the present author[4] arrived at a new interpretation of the ground state Bohr radius, a_B of a hydrogen (H) atom obtained from its ionization potential (I_H). It was shown that a_B, which is the distance between the proton and electron, is divided at the Golden point into two Golden sections, a_{e-} and a_{p+} pertaining to the electron (e^-) and proton (p^+), respectively. This interpretation was extended[5] to any atom (A) and the Golden sections of the Bohr radius are related as follows,

$$a_{B,A} = e/2\kappa I_1 = a_{e-} + a_{n+} \quad (1a)$$

$$a_{n+} = (a_{B,A}/\phi^2) \text{ and } a_{e-} = (a_{B,A}/\phi) = \phi a_{n+} \quad (1b,c)$$

where e is the charge, κ is the electrical permittivity of vacuum, $e/2\kappa = 7.1998$ Å.eV and $\phi = (1+5^{1/2})/2) = 1.618$ is the Golden ratio[6], also called The Divine Ratio.

1.2 Golden sections of the bond lengths, d(AA) as ionic radii.

The finding[4] that the bond length d(AA) between two atoms (A) of the same kind is divided at the Golden point into two Golden sections, $d(A^-)$ and $d(A^+)$, which form the anionic (A^-) and cationic (A^+) radii of atoms is expressed by the relations,

$$d(AA) = 2d(A) = d(A^-) + d(A^+) \quad (2a)$$

$$d(A^+) = d(AA)/\phi^2 \text{ and } d(A^-) = d(AA)/\phi = \phi d(A^+) \quad (2b,c)$$

where $d(A) = d(AA)/2$ is defined[7] as the covalent radius. *Note*: the symbol d is used here for radii since they are apportioned distances.

1.3 Golden ratio linking atomic and Bohr radii for elements of Groups 1A- 8A.

In another paper[8], it was shown that the various radii of atoms, including the covalent atomic radii, d(A) obtained from lattice constants and bond lengths d(AA), vary linearly with their Bohr radii, $a_{B,A}$. This observation was recently made[5] more precise, which led to the fundamental discovery that the Golden ratio links the atomic radii d(A) with the Bohr radii $a_{B,A}$, through the simple equation,

$$d(A) = K_\phi a_{B.A} \quad (3)$$

for the elements of Groups 1A - 8A, where K_ϕ is a simple function of the Golden ratio. By using the data for the first ionization potentials from[9] and the bond lengths, d(AA) from[10], the above equation

---

✎ **Raji Heyrovska (Correspondence)**
✉ rheyrovs@hotmail.com

was established and the data were presented in Table 1 in[5].

2. Present work: Golden ratio linking atomic and Bohr radii for elements of Groups 1B- 8B including Lanthanides and Actinides

The above work[5] has been extended in this paper to the elements of Groups 1B - 8B. It is shown that equation (3) holds for these elements as well. Table 1 presents the data for Group 1A - 8A elements, where the data for some elements (mentioned in the title of the Table 1) have been revised. Figure 1 shows the revised graph of $d(A)_{cal}$ versus d(A).

The data for the elements of Groups 1B - 8B have been assembled in Table 2 and those for the Lanthanides and Actinides are assembled in Table 3. The corresponding graph of $d(A)_{cal}$ versus d(A) can be seen in Figure 2. The Figures show the general validity of equation (3) for the elements of all the Groups 1 - 8 and that the Golden ratio is an important geometrical constant in the atomic structure of elements.

References

[1] Shechtman, D. http://www.nobelprize.org/nobel_prizes/chemistry/laureates/2011/shechtman-lecture.html

[2] Coldea, R., et al, Quantum criticality in an Ising chain: experimental evidence for emergent E_8 symmetry, *Science*, **327,** 177-180 (2010).

[3] Heyrovska, R., Structures at the atomic level of cobalt, zinc and lead niobates (with an Appendix: Atomic structure of cobalt niobate crystal) http://precedings.nature.com/documents/6059/version/2

[4] Heyrovska, R. The Golden ratio, ionic and atomic radii and bond lengths. *Molecular Physics*, **103,** 877 - 882 (2005); and the literature therein.

[5] Heyrovska, R. Atomic and Ionic Radii of Elements and Bohr Radii from Ionization Potentials are Linked Through the Golden Ratio. *International J. Sciences*, **2**, 1-4, Apr 2013; http://www.ijsciences.com/pub/pdf/V2-201303-019.pdf

[6] Livio, M. *The Golden Ratio: The Story of PHI, the World's Most Astonishing Number*, Broadway Books, NY, (2002).

[7] Pauling, L. *The Nature of the Chemical Bond*, Cornell Univ. Press, NY, (1960).

[8] Heyrovska, R. a) Direct dependence of covalent, van der Waals and valence shell radii of atoms on their Bohr radii for elements of Groups 1A - 8A. Ohtaki Memorial Lecture, 10th Eurasia Conference on Chemical Sciences, Manila, Philippines, 2008. *Philippine J. Sci.* **137,** 133-139, (2008); b) http://arxiv.org/abs/0708.1108v2

[9] http://physics.nist.gov/PhysRefData/IonEnergy/tblNew.html

[10] http://www.webelements.com/lithium/crystal_structure.html

Figure legends

Figure 1. Linear correlation of calculated values of atomic radii, $d(A)_{cal}$ with the observed values, d(A) = d(AA)/2 for elements of Groups 1A - 8A.

Figure 2. Linear correlation of calculated values of atomic radii, $d(A)_{cal}$ with the observed values, d(A) = d(AA)/2 for elements of Groups 1B - 8B including Lanthanides and Actinides (where data were available).

Figure 1

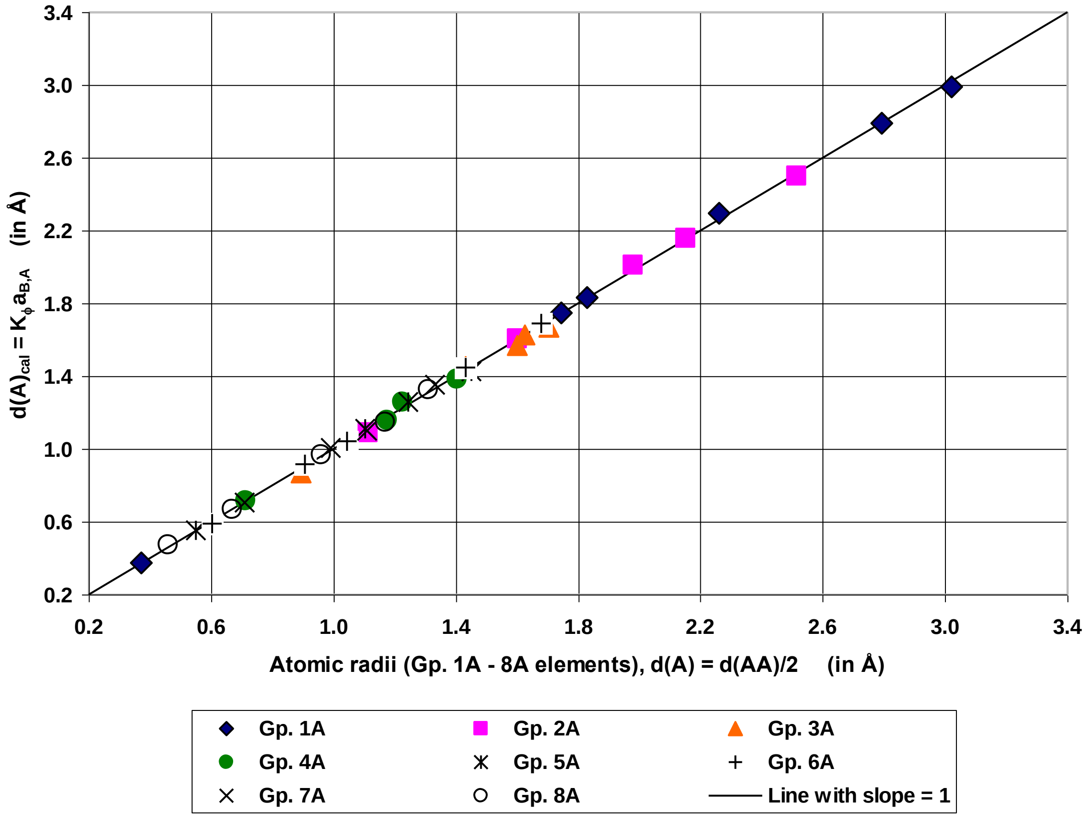

Figure 2

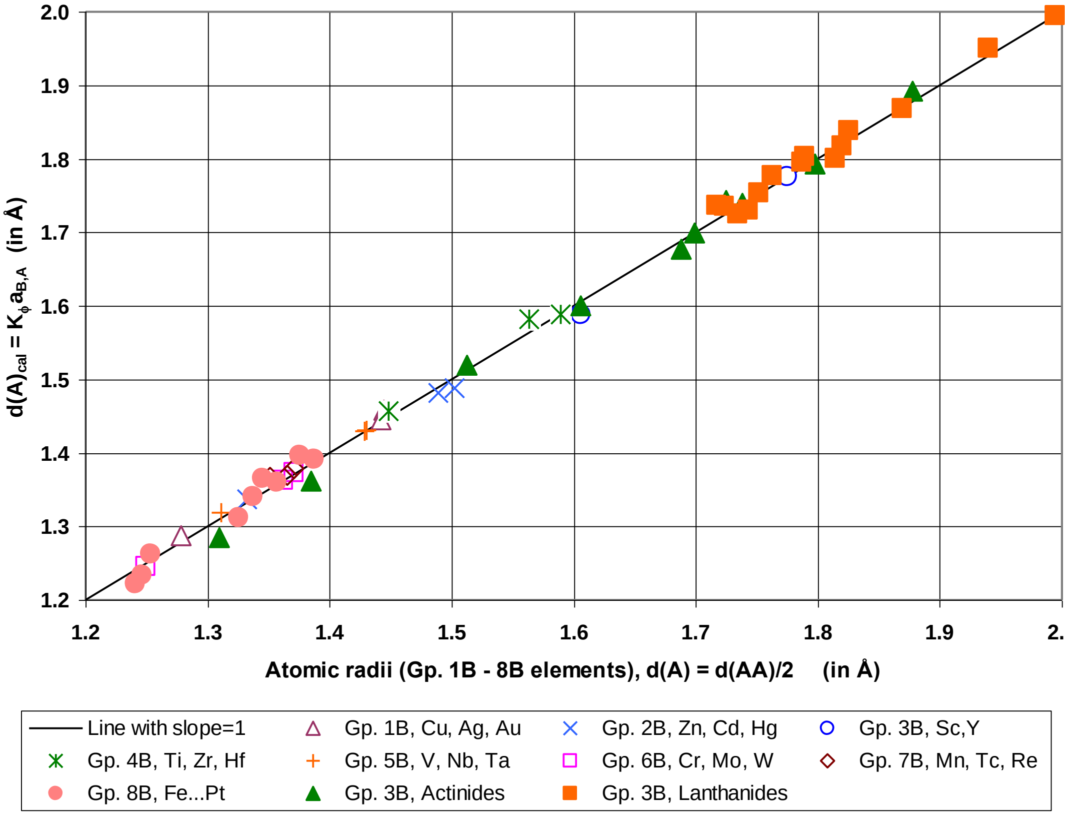
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
$d(A)_{cal} = K_{\phi} a_{B,A}$ (in Å)
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
Atomic radii (Gp. 1B - 8B elements), d(A) = d(AA)/2 (in Å)
Line with slope=1
△ Gp. 1B, Cu, Ag, Au
× Gp. 2B, Zn, Cd, Hg
○ Gp. 3B, Sc,Y
Gp. 4B, Ti, Zr, Hf
+ Gp. 5B, V, Nb, Ta
☐ Gp. 6B, Cr, Mo, W
◇ Gp. 7B, Mn, Tc, Re
● Gp. 8B, Fe...Pt
▲ Gp. 3B, Actinides
■ Gp. 3B, Lanthanides

Table 1. Covalent radii, d(A) = d(AA)/2 of atoms (A) of elements of Groups 1A - 8A, related to Bohr radii, $a_{B,A}$ (= e/2kI_1) from 1st ionization potentials, I_1 (in eV) through the Golden ratio, ϕ (= 1.618). Ionic radii, $d(A^-) = d(AA)/\phi$ and $d(A^+) = d(AA)/\phi^2$. $d(A)_{cal} = K_\phi a_{B,A}$. Radii $a_{e-} = a_{B,A}/\phi$ of the electron (e^-) and $a_{n+} = a_{B,A}/\phi^2$ of the nucleus (A_{n+}) of A. All radii are in Å.

* Atoms for which values have been revised from those in[5].

Gp.	**Atom**	I_1	$a_{B,A}$	a_{e-}	a_{n+}	**d(A)**	**$d(A^-)$**	**$d(A^+)$**	K_ϕ	K_ϕ =	**$d(A)_{cal}$**
1A	**H**	13.598	0.53	0.33	0.20	**0.37**	0.46	0.28	$1/2^{1/2}$	0.707	**0.37**
	Li	5.392	1.34	0.83	0.51	**1.75**	2.16	1.33	$\phi^2/2$	1.309	**1.75**
	Na	5.139	1.40	0.87	0.54	**1.83**	2.26	1.40	$\phi^2/2$	1.309	**1.83**
	K	4.341	1.66	1.03	0.63	**2.26**	2.80	1.73	$(\phi^2+1)/\phi^2$	1.382	**2.29**
	Rb	4.177	1.72	1.07	0.66	**2.79**	3.45	2.13	ϕ	1.618	**2.79**
	Cs	3.894	1.85	1.14	0.71	**3.02**	3.74	2.31	ϕ	1.618	**2.99**
2A	**Be,***	9.323	0.77	0.48	0.29	**1.11**	1.38	0.85	$(2)^{0.5}$	1.414	**1.09**
	Mg,*	7.646	0.94	0.58	0.36	**1.60**	1.98	1.22	$1+1/2^{1/2}$	1.707	**1.61**
	Ca,*	6.113	1.18	0.73	0.45	**1.98**	2.45	1.51	$1+1/2^{1/2}$	1.707	**2.01**
	Sr,*	5.695	1.26	0.78	0.48	**2.15**	2.66	1.64	$1+1/2^{1/2}$	1.707	**2.16**
	Ba	5.212	1.38	0.85	0.53	**2.51**	3.11	1.92	$(\phi^2+1)/2$	1.809	**2.50**
3A	**B**	8.298	0.87	0.54	0.33	**0.89**	1.11	0.68	1.00	1.000	**0.87**
	Al,*	5.986	1.20	0.74	0.46	**1.43**	1.77	1.09	$(1+2^{1/2})/2$	1.207	**1.45**
	Ga	5.999	1.20	0.74	0.46	**1.60**	1.98	1.22	$\phi^2/2$	1.309	**1.57**
	In	5.786	1.24	0.77	0.48	**1.63**	2.01	1.24	$\phi^2/2$	1.309	**1.63**
	Tl,*	6.108	1.18	0.73	0.45	**1.70**	2.11	1.30	$(2)^{0.5}$	1.414	**1.67**
4A	**C**	11.260	0.64	0.40	0.24	**0.71**	0.88	0.54	$(\phi^2+1)/2\phi$	1.118	**0.71**
	Si	8.152	0.88	0.55	0.34	**1.18**	1.45	0.90	$\phi^2/2$	1.309	**1.16**
	Ge	7.899	0.91	0.56	0.35	**1.23**	1.51	0.94	$(\phi^2+1)/\phi^2$	1.382	**1.26**
	Sn,*	7.344	0.98	0.61	0.37	**1.41**	1.74	1.07	$(2)^{0.5}$	1.414	**1.39**
5A	**N**	14.534	0.50	0.31	0.19	**0.55**	0.68	0.42	$(\phi^2+1)/2\phi$	1.118	**0.55**
	P	10.487	0.69	0.42	0.26	**1.11**	1.37	0.84	ϕ	1.618	**1.11**
	As,*	9.789	0.74	0.45	0.28	**1.25**	1.54	0.95	$1+1/2^{1/2}$	1.707	**1.26**
	Sb,*	8.608	0.84	0.52	0.32	**1.45**	1.79	1.11	$1+1/2^{1/2}$	1.707	**1.43**
6A	**O**	13.618	0.53	0.33	0.20	**0.60**	0.75	0.46	$(\phi^2+1)/2\phi$	1.118	**0.59**
	S,*	10.360	0.69	0.43	0.27	**1.04**	1.29	0.80	1.5	1.500	**1.04**
	Se,*	11.814	0.61	0.38	0.23	**0.91**	1.12	0.69	1.5	1.500	**0.91**
	Te	9.010	0.80	0.49	0.31	**1.43**	1.77	1.09	$(\phi^2+1)/2$	1.809	**1.45**
	Po,*	8.414	0.86	0.53	0.33	**1.68**	2.08	1.28	$3\phi^2/4$	1.964	**1.69**
7A	**F,***	17.423	0.41	0.26	0.16	**0.71**	0.88	0.54	$1+1/2^{1/2}$	1.707	**0.71**
	Cl	12.968	0.56	0.34	0.21	**0.99**	1.22	0.76	$(\phi^2+1)/2$	1.809	**1.00**
	Br	11.814	0.61	0.38	0.23	**1.11**	1.37	0.85	$(\phi^2+1)/2$	1.809	**1.10**
	I,*	10.451	0.69	0.43	0.26	**1.33**	1.65	1.02	$3\phi^2/4$	1.964	**1.35**
8A	**He**	24.587	0.29	0.18	0.11	**0.46**	0.57	0.35	ϕ	1.618	**0.47**
	Ne,*	21.565	0.33	0.21	0.13	**0.67**	0.83	0.51	2	2.000	**0.67**
	Ar	15.760	0.46	0.28	0.17	**0.96**	1.19	0.73	$\phi^3/2$	2.118	**0.97**
	Kr,*	14.000	0.51	0.32	0.20	**1.17**	1.45	0.89	$(5)^{0.5}$	2.236	**1.15**
	Xe,*	12.130	0.59	0.37	0.23	**1.31**	1.62	1.00	$(5)^{0.5}$	2.236	**1.33**

Table 2. Covalent radii, d(A) = d(AA)/2 of atoms (A) of elements of Groups 1B - 8B, related to Bohr radii, $a_{B,A}$ (= $e/2kI_1$) from 1st ionization potentials, I_1 (in eV) through the Golden ratio, ϕ (= 1.618). Ionic radii, $d(A^-) = d(AA)/\phi$ and $d(A^+) = d(AA)/\phi^2$. $d(A)_{cal} = K_\phi a_{B,A}$. Radii $a_{e-} = a_{B,A}/\phi$ of the electron (e^-) and $a_{n+} = a_{B,A}/\phi^2$ of the nucleus (A_{n+}) of A. All radii are in Å.

Gp.	**Atom**	**I_1**	**$a_{B,A}$**	**a_{e-}**	**a_{n+}**	**d(A)**	**$d(A^-)$**	**$d(A^+)$**	**K_ϕ**	**K_ϕ =**	**$d(A)_{cal}$**
1B	**Cu**	7.7264	0.93	0.58	0.36	**1.28**	1.58	0.98	$(\phi^2+1)/\phi^2$	1.382	**1.29**
	Ag	7.5762	0.95	0.59	0.36	**1.44**	1.79	1.10	$\phi^2/(1+2^{-1/2})$	1.534	**1.46**
	Au	9.2255	0.78	0.48	0.30	**1.44**	1.78	1.10	$2^{0.5}\phi^2$	1.851	**1.44**
2B	**Zn**	9.3942	0.77	0.47	0.29	**1.33**	1.65	1.02	$\phi^2/1.5$	1.745	**1.34**
	Cd	8.9938	0.80	0.49	0.31	**1.49**	1.84	1.14	$2^{0.5}\phi^2$	1.851	**1.48**
	Hg	10.4375	0.69	0.43	0.26	**1.50**	1.86	1.15	$4\phi/3$	2.157	**1.49**
3B	**Sc**	6.5615	1.10	0.68	0.42	**1.61**	1.99	1.23	$2\phi^2/(\phi^2+1)$	1.447	**1.59**
	Y	6.2173	1.16	0.72	0.44	**1.78**	2.19	1.36	$\phi^2/(1+2^{-1/2})$	1.534	**1.78**
4B	**Ti**	6.8281	1.05	0.65	0.40	**1.45**	1.79	1.11	$(\phi^2+1)/\phi^2$	1.382	**1.46**
	Zr	6.6339	1.09	0.67	0.41	**1.59**	1.96	1.21	$(\phi^2+1)\phi/4$	1.463	**1.59**
	Hf	6.8251	1.05	0.65	0.40	**1.56**	1.93	1.19	1.50	1.500	**1.58**
5B	**V**	6.7462	1.07	0.66	0.41	**1.31**	1.62	1.00	$2/\phi$	1.236	**1.32**
	Nb	6.7589	1.07	0.66	0.41	**1.43**	1.77	1.09	$3/(2\phi-1)$	1.342	**1.43**
	Ta	7.5496	0.95	0.59	0.36	**1.43**	1.77	1.09	1.50	1.500	**1.43**
6B	**Cr**	6.7665	1.06	0.66	0.41	**1.25**	1.54	0.95	$\phi^2/(2\phi-1)$	1.171	**1.25**
	Mo	7.0924	1.02	0.63	0.39	**1.36**	1.68	1.04	$3/(2\phi-1)$	1.342	**1.36**
	W	7.8640	0.92	0.57	0.35	**1.37**	1.69	1.05	1.50	1.500	**1.37**
7B	**Mn**	7.4340	0.97	0.60	0.37	**1.37**	1.69	1.04	$(2)^{0.5}$	1.414	**1.37**
	Tc	7.2800	0.99	0.61	0.38	**1.35**	1.67	1.03	$(\phi^2+1)/\phi^2$	1.382	**1.37**
	Re	7.8335	0.92	0.57	0.35	**1.37**	1.69	1.05	1.50	1.500	**1.38**
8B	**Fe**	7.9024	0.91	0.56	0.35	**1.24**	1.53	0.95	$3/(2\phi-1)$	1.342	**1.22**
	Co	7.8810	0.91	0.56	0.35	**1.25**	1.55	0.96	$(\phi^2+1)/\phi^2$	1.382	**1.26**
	Ni	7.6399	0.94	0.58	0.36	**1.25**	1.54	0.95	$\phi^2/2$	1.309	**1.23**
	Ru	7.3605	0.98	0.60	0.37	**1.33**	1.64	1.01	$3/(2\phi-1)$	1.342	**1.31**
	Rh	7.4589	0.97	0.60	0.37	**1.35**	1.66	1.03	$(2)^{0.5}$	1.414	**1.36**
	Pd	8.3369	0.86	0.53	0.33	**1.38**	1.70	1.05	ϕ	1.618	**1.40**
	Os	8.4382	0.85	0.53	0.33	**1.34**	1.65	1.02	$3\phi^2/5$	1.571	**1.34**
	Ir	8.9670	0.80	0.50	0.31	**1.36**	1.68	1.04	$2\phi^3/5$	1.694	**1.36**
	Pt	8.9588	0.80	0.50	0.31	**1.39**	1.72	1.06	1.732	1.732	**1.39**

Table 3. Covalent radii, d(A) = d(AA)/2 of atoms (A) of elements of Gp. 3B: La-Lu and Ac-Cf, related to Bohr radii, $a_{B,A}$ (= $e/2kI_1$) from 1st ionization potentials, I_1 (in eV) through the Golden ratio ϕ (= 1.618). Ionic radii, $d(A^-) = d(AA)/\phi$ and $d(A^+) = d(AA)/\phi^2$. $d(A)_{cal} = K_\phi a_{B,A}$. Radii $a_{e-} = a_{B,A}/\phi$ of the electron (e^-) and $a_{n+} = a_{B,A}/\phi^2$ of the nucleus (A_{n+}) of A. All radii are in Å.

Gp.	**Atom**	**I_1**	**$a_{B,A}$**	**a_{e-}**	**a_{n+}**	**d(A)**	**$d(A^-)$**	**$d(A^+)$**	**K_ϕ**	**K_ϕ =**	**$d(A)_{cal}$**
3B	**La**	5.5769	1.29	0.80	0.49	**1.87**	2.31	1.43	$2\phi^2/(\phi^2 +1)$	1.447	**1.87**
	Ce	5.5387	1.30	0.80	0.50	**1.83**	2.26	1.39	$(2)^{0.5}$	1.414	**1.84**
	Pr	5.4730	1.32	0.81	0.50	**1.82**	2.25	1.39	$(\phi^2+1)/\phi^2$	1.382	**1.82**
	Nd	5.5250	1.30	0.81	0.50	**1.81**	2.24	1.39	$(\phi^2+1)/\phi^2$	1.382	**1.80**
	Sm	5.6437	1.28	0.79	0.49	**1.79**	2.21	1.37	$(2)^{0.5}$	1.414	**1.80**
	Eu	5.6704	1.27	0.78	0.48	**1.99**	2.47	1.52	$3\phi^2/5$	1.571	**1.99**
	Gd	6.1498	1.17	0.72	0.45	**1.79**	2.21	1.36	$\phi^2/(1+2^{-1/2})$	1.534	**1.80**
	Tb	5.8638	1.23	0.76	0.47	**1.76**	2.18	1.35	$2\phi^2/(\phi^2 +1)$	1.447	**1.78**
	Dy	5.9389	1.21	0.75	0.46	**1.75**	2.17	1.34	$2\phi^2/(\phi^2 +1)$	1.447	**1.75**
	Ho	6.0215	1.20	0.74	0.46	**1.74**	2.15	1.33	$2\phi^2/(\phi^2 +1)$	1.447	**1.73**
	Er	6.1077	1.18	0.73	0.45	**1.73**	2.14	1.32	$(\phi^2+1)\phi/4$	1.463	**1.72**
	Tm	6.1843	1.16	0.72	0.44	**1.72**	2.13	1.32	$2(2\phi-1)/3$	1.491	**1.74**
	Yb	6.2542	1.15	0.71	0.44	**1.94**	2.40	1.48	$2\phi^3/5$	1.694	**1.95**
	Lu	5.4259	1.33	0.82	0.51	**1.72**	2.12	1.31	$\phi^2/2$	1.309	**1.74**
3B	**Ac**	5.3807	1.34	0.83	0.51	**1.88**	2.32	1.43	$(2)^{0.5}$	1.414	**1.89**
	Th	6.3067	1.14	0.71	0.44	**1.80**	2.22	1.37	$3\phi^2/5$	1.571	**1.79**
	Pa	5.89	1.22	0.76	0.47	**1.61**	1.99	1.23	$\phi^2/2$	1.309	**1.60**
	U	6.1939	1.16	0.72	0.44	**1.39**	1.71	1.06	$\phi^2/(2\phi-1)$	1.171	**1.36**
	Np	6.2657	1.15	0.71	0.44	**1.31**	1.62	1.00	$(\phi^2+1)/2\phi$	1.118	**1.28**
	Pu	6.026	1.19	0.74	0.46	**1.51**	1.87	1.16	$(\phi)^{0.5}$	1.272	**1.52**
	Am	5.9738	1.21	0.74	0.46	**1.73**	2.13	1.32	$2\phi^2/(\phi^2 +1)$	1.447	**1.74**
	Cm	5.9914	1.20	0.74	0.46	**1.74**	2.15	1.33	$2\phi^2/(\phi^2 +1)$	1.447	**1.74**
	Bk	6.1979	1.16	0.72	0.44	**1.70**	2.10	1.30	$(\phi^2+1)\phi/4$	1.463	**1.70**
	Cf	6.2817	1.15	0.71	0.44	**1.69**	2.09	1.29	$(\phi^2+1)\phi/4$	1.463	**1.68**