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Golden Ratio Based Fine Structure Constant and Rydberg Constant for Hydrogen Spectra

Raji Heyrovska[1]

[1]Na Stahlavce 6, 16000 Praha 6, Czech Republic
[1](Dedicated to Niels Bohr on the occasion of the centenary of his paper on hydrogen)

Abstract: The surprising discovery that Bohr radius is divided at the Golden point into two sections pertaining to the electron and proton due to electrostatic reasons led to the general finding that it is a geometrical constant in atomic and ionic radii, bond lengths and bond angles. In an earlier article, the fine structure constant was also interpreted in terms of the Golden ratio. Here experimental data on wavelengths of hydrogen spectral lines have been used to evaluate the Rydberg constant, fine structure constant and Sommerfeld's relativity factor and are all shown to be simple functions of the Golden ratio.

Keywords: Hydrogen spectra, Fine structure constant, Golden ratio, Relativity factor, Rydberg constant, Rydberg formula, Bohr radius

1. Introduction

1.1 Golden ratio as a geometrical constant in the dimensions of atoms

Some years ago, the author made a new suggestion[1] that the ground state ionization potential (I_H) of hydrogen (H), could be considered as the difference between the potentials, (I_p) and (I_e) of the oppositely charged proton (p^+) and electron (e^-), necessary to pull them apart against their electrostatic interaction. This is expressed by the equation,

$$I_H = e/2\kappa a_B = (e/2\kappa)[(1/a_{B,p}) - (1/a_{B,e})] \quad (1)$$

where e is the electric charge, κ is the electric constant and a_B is the Bohr radius (= distance between the proton and electron) which is the sum $a_{B,p} + a_{B,e}$, of the two sections pertaining to the proton (p^+) and electron (e^-). On using the data from[2], $e/2\kappa$ = 7.19982 Å.eV and I_H = 13.5984 eV, the value of $a_B = e/2\kappa I_H$ = 0.052946 nm.

From equation (1), one obtains the following successive relations,

$$1/a_B = 1/(a_{B,p} + a_{B,e}) = (1/a_{B,p}) - (1/a_{B,e}) \quad (2)$$

$$(a_{B,e}/a_{B,p})^2 - (a_{B,e}/a_{B,p}) - 1 = 0 \quad (3)$$

$$(a_B/a_{B,e}) = (a_{B,e}/a_{B,p}) = (1 + 5^{1/2})/2 = \phi = 1.618 \quad (4)$$

$$a_{B,e} = a_B/\phi = 0.03272 \text{ nm and } a_{B,p} = a_B/\phi^2 = 0.02022 \text{ nm} \quad (5a,b)$$

where ϕ is called the Golden ratio or The Divine ratio[3]. It was thus a surprise to find[1] for the first time that the Bohr radius is divided into two unique sections at the point of electrical neutrality, which is the Golden point. The Golden ratio[3], which manifests itself in many spontaneous creations of *Nature* was thus found to originate right in the core of atoms[1,4].

It was soon demonstrated[1] that the covalent bond lengths between atoms of the same kind are also divided at the Golden point into two sections which form the cationic and anioinc radii. The additivity of atomic and ionic radii in the bond lengths in many inorganic, organic and biological molecules and in aqueous media were then established[4-8]. Recently, atomic radii of all elements obtained from the lattice parameters and bond lengths were shown[9,10] to be related to the respective Bohr radii obtained from first ionization potentials through simple functions of the Golden ratio. Bond angles in simple molecules like water, carbon dioxide, sulfur dioxide, etc. were also interpreted[11] in terms of the Golden ratio.

1.2. Golden ratio based fine structure constant as a mathematical constant

The fine structure constant[12] (α), which has the value[2], 1/137.036 = 0.0072974, is the ratio of the Compton wavelength ($\lambda_{C,H}$) to the de Broglie wavelength ($\lambda_{dB} = 2\pi a_B$), and is also equal to the ratio of twice the de Broglie wavelength to the Rydberg wavelength (= $\lambda_H = R_H^{-1}$, where R_H is the Rydberg constant),

Raji Heyrovska (Correspondence)
rheyrovs@hotmail.com

$\alpha = \lambda_{C,H}/\lambda_{dB} = 2\lambda_{dB}/\lambda_H = 0.0072974 = 2.627/360$ (6)

As $\lambda_{dB} = 2\pi a_B = 0.3327$ nm is the circumference of the Bohr circle, the Compton wavelength is the length of a small arc, $\lambda_{C,H} = \alpha\lambda_{dB} = 0.00243$ nm on the Bohr circle, which subtends an angle 2.627^o at the center of the Bohr circle. Similarly, $2\lambda_{dB} = 0.6653$ nm is equivalent to the length of an arc of a circle subtending an angle 2.627^o at the center of a circle of with circumference, $\lambda_H = 91.1756$ nm and radius = $\lambda_H/2\pi = 14.511$ nm.
On noticing[12] the closeness of the fine structure constant, $\alpha = 2.627/360$ to the ratio of the angles, $\phi^2/360 = 2.618/360$ the author suggested that the small difference, $2.627^0 - 2.618^0 = 0.009^0$ could be due to the Sommerfeld's relativity correction factor[13],

$\alpha - (1- \gamma)/\gamma = (\phi^2/360) = \alpha_o$ (7)

where $\gamma = (1 - \alpha^2)^{1/2}$. For $\alpha = 0.0072974$, $(1- \gamma)/\gamma$ ($= \alpha_{rel} = 0.0096/360 = 0.0000267$) and $\alpha - \alpha_o = 0.0072974 – 0.0072723 = 0.0000251$.
It was also pointed out[12] that the ratio $360/\phi^2 = 137.508$, which is a Golden section of 360, differs from the inverse fine structure constant[2] ($\alpha^{-1} = 137.036$) by $2/\phi^3 = 0.472$ probably due to the difference in the g-factors for the electron and proton, g_e and g_p arising from the magnetic momenta of the two particles. Thus,

$360/\phi^2 - \alpha^{-1} = (g_p - g_e)/(g_p + g_e) = 2(a_{B,e} - a_{B,p})/a_B = 2/\phi^3 = 0.472$ (8a)

$\alpha^{-1} = (360 - 2/\phi)/\phi^2 = 137.036$ (8b)

$\alpha = \phi^2/(360 - 2/\phi) = 0.0072974$ (8c)

As per equation (8b), where $360 - 2/\phi = 358.764^o$, the Golden ratio based value of $\alpha^{-1} = 137.0356$ rounds up to the value, 137.036 in[12]. All details and a descriptive figure can be found in[12]. Thus, equations (8b) and (8c) represent the fine structure constant as a mathematical constant in terms of the Golden ratio.

2. Present work

2.1. The Rydberg constant evaluated from measured wavelengths of hydrogen spectrum.

In this work, the measured wavelengths[14] of the hydrogen spectrum (see column 2, Table 1) have been used to evaluate the inverse Rydberg constant, $R_H^{-1} = \lambda_H$ (which can be called the Rydberg wavelength) using the Rydberg equation,

$1/\lambda = R_H[(1/n_1^2) - (1/n_2^2)]$ (9)

$R_H = 1/\lambda_H = \alpha/4\pi a_B$ (10)

where n_1 and n_2 ($> n_1$) are the principal quantum numbers and $a_B = e/2\kappa I_H = 0.052946$ nm from equation (1). On rewriting Rydberg equation (9) in the form,

$\lambda[(1/n_1^2) - (1/n_2^2)] = \lambda_H = (4\pi a_B/\alpha)$ (11)

one can see that the product on the left side must be a constant equal to the Rydberg wavelength, λ_H, for all wavelengths, λ. These products corresponding to all the wavelengths (in column 2) are given in column 6 in Table 1.
For $a_B = 0.052946$ nm and $\alpha^{-1} = 137.036$, $\lambda_H = (4\pi a_B/\alpha) = 91.175$ nm. It can be seen from the products in column 6 that this value of λ_H is nearly reproduced by the Lyman series, but the values for Balmer and Paschen series are slightly lower. For the wavelengths for the Bracket and Pfund series, λ_H values are not the same. A graph of $1/\lambda$ vs $[(1/n_1^2) - (1/n_2^2)]$ (excluding the last two series) in Figure 1 here gives the slope, $R_H = 1.09670 \times 10^7$ m^{-1}, and its reciprocal, $\lambda_H = 91.186$ nm. This least squares value is given in the top of column 6.

2.2. The fine structure constant, relativity factor and Golden ratio derived from the Rydberg constant

The values of the fine structure constant, α obtained as the ratio, $4\pi a_B/\lambda_H$, as per equation (11), using the λ_H values in column 6, are given in column 7. But for the Lyman series, these values also differ slightly from the value, $1/137.036 = 0.0072974$.
In column 8 are given the values of $(1- \gamma)/\gamma = \alpha_{rel}$, where $\gamma = (1 - \alpha^2)^{1/2}$ were calculated from the α values in column 7, as per equation (7). The difference, $\alpha - \alpha_{rel} = \alpha_o$ is given in column 9. Column 10 gives the calculated values of $360\alpha_o = \phi^2$. It can be seen that these values of ϕ^2, evaluated from the Rydberg wavelength in column 6, are close to 2.618, in accordance with equation (7).

2.3. The Rydberg equation in terms of the Goden ratio

On using the Golden ratio based equation (8b) for $\alpha = \phi^2/(360 - 2/\phi)$, the Rydberg equation (9) can be written in terms of the Golden ratio as,

$1/\lambda = [\phi^2/(360 - 2/\phi)(4\pi a_B)][(1/n_1^2) - (1/n_2^2)]$ (12a)

$R_H = (\alpha/4\pi a_B) = [\phi^2/(360 - 2/\phi)(4\pi a_B)]$ (12b)

where $a_B/\phi^2 = a_{B,p} = 0.02022$ nm (see equation 5b) is the Golden section of the Bohr radius pertaining to the proton and $2\pi a_{B,p} = \lambda_{dB,p} = \lambda_{dB}/\phi^2$ is the circumference of the circle with radius, $a_{B,p}$ with the proton at the center. λ_{dB}/ϕ^2 is also the Golden arc of the Bohr circle, subtending an angle, $360/\phi^2$ at the center.

Summary: The Golden ratio based fine structure constant and Rydberg formula are given by equations (8b) and (12a,b) respectively.

References

[1] Heyrovska, R. The Golden ratio, ionic and atomic radii and bond lengths. *Molecular Physics*, **103,** 877 - 882 (2005); and the literature therein.
[2] http://physics.nist.gov/cuu/Constants/
[3] Livio, M. *The Golden Ratio: The Story of PHI, the World's Most Astonishing Number*, Broadway Books, NY, (2002).
[4] Heyrovska, R.The Golden ratio in the Creations of Nature Arises in the Architecture of Atoms and Ions. Chapter 12 in Book: *"Innovations in Chemical Biology"*, Editor: Bilge Sener, Springer.com, January (2009).
[5] Heyrovska, R. and Narayan, S. Atomic Structures of Molecules Based on Additivity of Atomic and/or Ionic Radii. *AIP Conference Proceedings* **1119**, 216 (2009); Nature Precedings http://precedings.nature.com/documents/3292/version/1
[6] Heyrovska, R.Dependence of the Length of the Hydrogen Bond on the Covalent and Cationic Radii of Hydrogen, and Additivity of Bonding Distances. *Chemical Physics Letters*, **432,** 348 - 351 (2006).
[7] Heyrovska, R. Dependences of Molar Volumes in Solids, Partial Molal and Hydrated Ionic Volumes of Alkali Halides on Covalent and Ionic Radii and the Golden Ratio. *Chemical Physics Letters*, **436**, 287 - 293 (2007).
[8] Heyrovska, R. Golden Sections of Interatomic Distances as Exact Ionic Radii and Additivity of Atomic and Ionic Radii in Chemical Bonds. arXiv:0902.1184 [pdf]
[9] Heyrovska, R. Atomic and Ionic Radii of Elements and Bohr Radii from Ionization Potentials are Linked Through the Golden Ratio. *International J. Sciences*., Vol 2, 82-92, Issue-Mar-2013. http://www.ijsciences.com/pub/pdf/V2-201303-19.pdf
[10] Heyrovska, R. Bond lengths, Bond angles and Bohr Radii from Ionization Potentials Related via the Golden Ratio for H_2^+, O_2, O_3, H_2O, SO_2, NO_2 and CO_2. *International J. Sciences*., Vol 2, 1-4, Issue-Apr-2013, http://www.ijsciences.com/pub/pdf/V2-201304-08.pdf
[11] Heyrovska, R. Atomic, Ionic and Bohr Radii Linked via the Golden Ratio for Elements Including Lanthanides and Actinides. *International J. Sciences*., Vol 2, 63-68, Issue-Apr-2013, http://www.ijsciences.com/pub/pdf/V2-201304-18.pdf
[12] Heyrovska, R. Fine-structure Constant, Anomalous Magnetic Moment, Relativity Factor and the Golden Ratio that Divides the Bohr Radius. http://arxiv.org/ftp/physics/papers/0509/0509207.pdf
[13] Ruark, A. E. and Urey, H. C. *Atoms, molecules and quanta*, Volume I, (Dover Publications, New York, 1964), p. 134.
[14] http://hyperphysics.phy-astr.gsu.edu/Hbase/tables/hydspec.html#c1

Figure Legend.

Figure 1. Linear dependence of **(**1/λ) on $[(1/n_1^2) - (1/n_2^2)]$ for the experimental data in[14] for the Lymann, Balmer and Paschen series.

Figure 1.

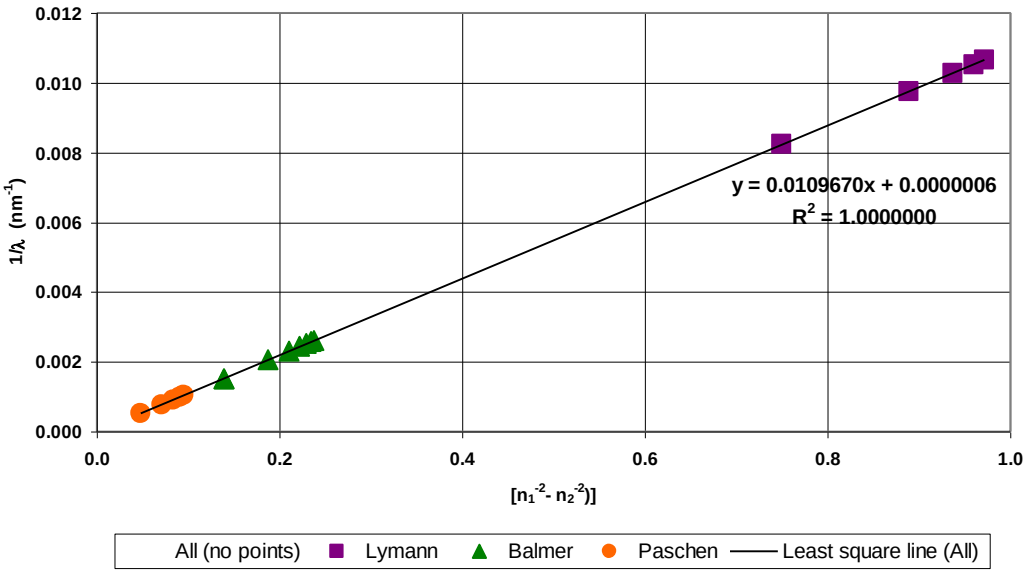

Table 1. Measured wavelengths (λ) of hydrogen spectra[14], Rydberg wavelength (λ_H), de Broglie wavelength ($\lambda_{dB} = 2\pi a_B$), fine structure constant (α), relativity factor [$(1-\gamma)/\gamma$] and square of the Golden ratio (ϕ^2).

					$R_H^{-1} = \lambda_H =$	$2\lambda_{dB}/\lambda_H =$	$(1-\gamma)/\gamma =$	$\alpha - \alpha_{rel} =$	$360\alpha_o =$
λ (nm)	**n_2**	**n_1**	**$1/\lambda$**	**$(n_1^{-2} - n_2^{-2})$**	**$\lambda(n_1^{-2} - n_2^{-2})$**	**α**	**α_{rel}**	**α_o**	**ϕ^2**
Least sq. value (this row):					**91.1826**	**0.0072968**	**0.00002662**	**0.0072701**	**2.6173**
Lymann									
93.782	6	1	0.010663	0.9722	91.1769	0.0072972	0.0000266	0.0072706	2.6174
94.976	5	1	0.010529	0.9600	91.1770	0.0072972	0.0000266	0.0072706	2.6174
97.254	4	1	0.010282	0.9375	91.1756	0.0072973	0.0000266	0.0072707	2.6175
102.583	3	1	0.009748	0.8889	91.1849	0.0072966	0.0000266	0.0072700	2.6172
121.566	2	1	0.008226	0.7500	91.1745	0.0072974	0.0000266	0.0072708	2.6175
Balmer									
383.5384	9	2	0.002607	0.2377	91.1496	0.0072994	0.0000266	0.0072728	2.6182
388.9049	8	2	0.002571	0.2344	91.1496	0.0072994	0.0000266	0.0072728	2.6182
397.0072	7	2	0.002519	0.2296	91.1496	0.0072994	0.0000266	0.0072728	2.6182
410.174	6	2	0.002438	0.2222	91.1498	0.0072994	0.0000266	0.0072728	2.6182
434.047	5	2	0.002304	0.2100	91.1499	0.0072994	0.0000266	0.0072727	2.6182
486.133	4	2	0.002057	0.1875	91.1499	0.0072994	0.0000266	0.0072727	2.6182
656.272	3	2	0.001524	0.1389	91.1489	0.0072995	0.0000266	0.0072728	2.6182
656.2852	3	2	0.001524	0.1389	91.1507	0.0072993	0.0000266	0.0072727	2.6182
Paschen									
954.62	8	3	0.001048	0.0955	91.1530	0.0072991	0.0000266	0.0072725	2.6181
1004.98	7	3	0.000995	0.0907	91.1546	0.0072990	0.0000266	0.0072724	2.6181
1093.8	6	3	0.000914	0.0833	91.1500	0.0072994	0.0000266	0.0072727	2.6182
1281.81	5	3	0.00078	0.0711	91.1509	0.0072993	0.0000266	0.0072727	2.6182
1875.01	4	3	0.000533	0.0486	91.1463	0.0072997	0.0000266	0.0072730	2.6183
Bracket									
2630	6	4	0.00038	0.0347	91.3194	0.0072858	0.0000266	0.0072592	2.6133
4050	5	4	0.000247	0.0225	91.1250	0.0073014	0.0000266	0.0072747	2.6189
Pfund									
7400	6	5	0.000135	0.0122	90.4444	0.0073563	0.0000265	0.0073298	2.6387