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A Dynamic Approach to Money Supply

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Abstract: In this paper, we present the mechanism of money supply from a dynamic perspective, in which the behaviors of the sectors involved in the process of money creation and the interplay among them are taken into account. Specially, we introduce households' withdrawals of deposit and firms' repayments to loan, which are ignored in the conventional statement of money creation process. By deriving and analyzing the equilibrium solution to the dynamic equations which characterize the process, we can discuss the corresponding influence of each sector on the money supply.

Introduction

Money is the blood of economy. A healthy money supply channel is an indispensable part to the maintaining and flourishing of the economy. To revive the economy from the devastating financial crisis, many countries turn to remedies that amplify the supply of money, which are also known as "quantitative easing" policy measures. The effect, however, is debatable. Actually, the process of money supply is systematic and complex, in which both banks and non-bank sectors play a role.

There are extensive literatures on the mechanism of money supply. C. A. Phillips argues that if banks do not retain excess reserves and the public only hold the demand deposits, no currency and no time deposits, the increase of reserves will lead to the increase of loans as well as the demand deposits [1]. The change in demand deposits is determined by the change in reserves and the inverse of required reserves ratio--the "textbook money multiplier". However, in reality, the banks do retain excess reserves to provide enhanced liquidity and the public hold their wealth not only in the form of demand deposit but also in the form of currencies. P. Samuelson takes both facts into consideration, measuring them with "excess reserves ratio"and "cash leakage ratio".Then the money multiplier will be formulated as the inverse of the sum of required reserves ratio, excess reserves ratio and currency leakage ratio [2].

P. Cagan analyzes the ratio of currency to the money supply, which fluctuated dramatically during 1875-1955 in America [3]. Then he proposes six decisive factors of the ratio. J. Ahrensdorf and S. Kanesathasan have performed an empirical analysis to test the effects of changes in currency, reserve ratios on the money supply. They also distinguish between the contribution of monetary authorities on the changes in the money supply and that of the private sector [4]. R. L. Teigen highlights the role of banking system in determining the money supply. He derives an aggregate money-supply function to segregate the exogenous and endogenous determinants on the money stock [5].

M. Friedman and A.Schwartz have performed a comprehensive study on this issue [6]. They develop a more detailed money multiplier model based on three proximate determinants, which are dependent on three sectors. It is also pointed out that the three factors have linkages with each other. Each one of them is jointly determined by the three sectors, rather than by a single sector respectively. P. Cagan carries out a deep and systematic research on the determinants and effects of the changes in the stock of money with the 1875-1960 data of America [7]. J.Jordan extends the analysis with a more complicated model by taking the differences between the member banks and the nonmember banks,

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as well as the fact that different kinds of deposits are subject to different reserve requirements [8].

Different from the model proposed in [6], K.Brunner and A.Meltzer create two money supply functions based on linear hypothesis and nonlinear hypothesis respectively. It uses statistical methods to have a regression which shows the multipliers of all the determinants of money stock, rather than for a single monetary base [9]. Compared with F-S's conceptual money multiplier, this method provides an empirical insight.

Some reformulations of money multiplier are made. C.Bourne proposes a process-oriented formulation of money multiplier with the case of Jamaica, which shows that in a small, open economy, the money market is demand centered [10]. A.W.A.McClean analyzes the properties of Bourne's model and acknowledges that the money multiplier analysis should be replaced to better accommodate the fact [11]. P.He,L.X.Huang, and R.Wright develop a new theory of money and banking based on the old story,then derives a money multiplier in a microfounded version [12]. M.Berardipresents an alternative and dynamic approach with the consideration of heterogeneous agents and their interactions [13].

It should be mentioned that economists have achieved fruitful results in understanding the mechanisms of the process of money supply. However, the quasi-static view adopted by the majority of the above works fails to capture the dynamics flows of the process as well as the explicit roles of different economic sectors. This paper, instead, provides a dynamic approach to understand the process of money supply and the determinants of money multiplier by analyzing both the money stocks and the in-and-out flows of the economy which are created and driven by the economic behaviors of different sectors including central and commercial banks, households and firms. With the established stock-flow relations, we not only present a dynamic analysis of the money creation process but also specifically compare and illustrate on the consequences of the sectors' behaviors in extreme cases such as debt crises.

A CONVENTIONAL APPROACH TO MONEY SUPPLY

In the conventional statement of money supply, three sectors participated in this process: central bank, commercial banks and non-bank public.

The central bank issues monetary base to public, then the public retains a part in the form of currencies, saving the rest. Commercial banks hand over a part of the deposits as required reserves to the central bank. Then they lend the rest excess reserves to public. Public repeats. Eventually, money supply will be a multiple of monetary base, which is the "money multiplier". The traditional model describes a quasi-static process, which cannot reflect the change of money supply in a dynamic view.

The base model can be depicted as follows. Monetary base MB is composed of currencies C and reserves R.The reserves can be further divided into required reserves RR and excess reserves ER. That is

$$MB = C + RR + ER \qquad (1)$$

These three components can be described respectively in terms of deposits D. As a result, Equation (1) can be rewritten as

$$MB = (c + r + e) \times D, \qquad (2)$$

where c is currency-deposit ratio , r is the required reserves ratio, and e is the excess reserves ratio.

Money supply M is composed of currencies and deposits, that is

$$M = C + D. \qquad (3)$$

Substituting (2) into (3), we have the money multiplier

$$m = \frac{M}{MB} = \frac{1+c}{c+r+e}. \qquad (4)$$

The conventional one mainly describes the money supply process conceptually without modelling the behavior of each sector. Regarding it as a quasi-static result, it derives a money supply within a money multiplier framework. Actually,this orthodox money multiplier is a poor description of money creation process.

ANALTERNATIVE APPROACH TO MONEY SUPPLY

Money creation is a systematic and dynamic process. Generally, the following four sectors are involved inthis process: the central bank, commercial banks, households and firms. The last two is sometimes combined into one sectorwhich is called "non-bank public" as given above. Each of them plays a key but different role.

The central bank issues the monetary base, and takes up the required reserves from commercial banks. Households save their money to commercial banks, and withdraw them when they need money. Their savings are called deposits. Firms can borrow money from commercial banks, and repay them when the loan is due.It follows that the commercial banks play a core role in the money creation process. They accept deposits from households and hold reserves required by the central bank or as a buffer for abrupt withdrawals. They also make loans to firms and get the matured ones back.

The money creation starts as central bank issues monetary base, or high-powered money, to households. Households hold a part as currencies for their need in transactions, then save the rest to commercial banks, through which the high-powered money is transformed to be reserves.Commercial banks accept deposits from households,and then hand over a portion to the central bank as required reserves. With the excess reserves, theycan loan some to firmsoncethey face demand ofborrowing. Simultaneously, an equivalent amount of deposits are created by banks and held by firms. So this is the vital action in the process of money creation.

However, this is not a complete story of money creation. Actually, besides saving and borrowing, households can withdraw money from banks when they have a demand for currency, firms must repay the loans at their due time. These behaviors operate in the opposite direction as saving and borrowing do and exert an apparent influence on money supply. The introduction of these two behaviors is one practical improvement for the conventional money creation process.

The mechanism mentioned above is an exhaustive description of the process of money creation, which can be illustrated as the following chart.

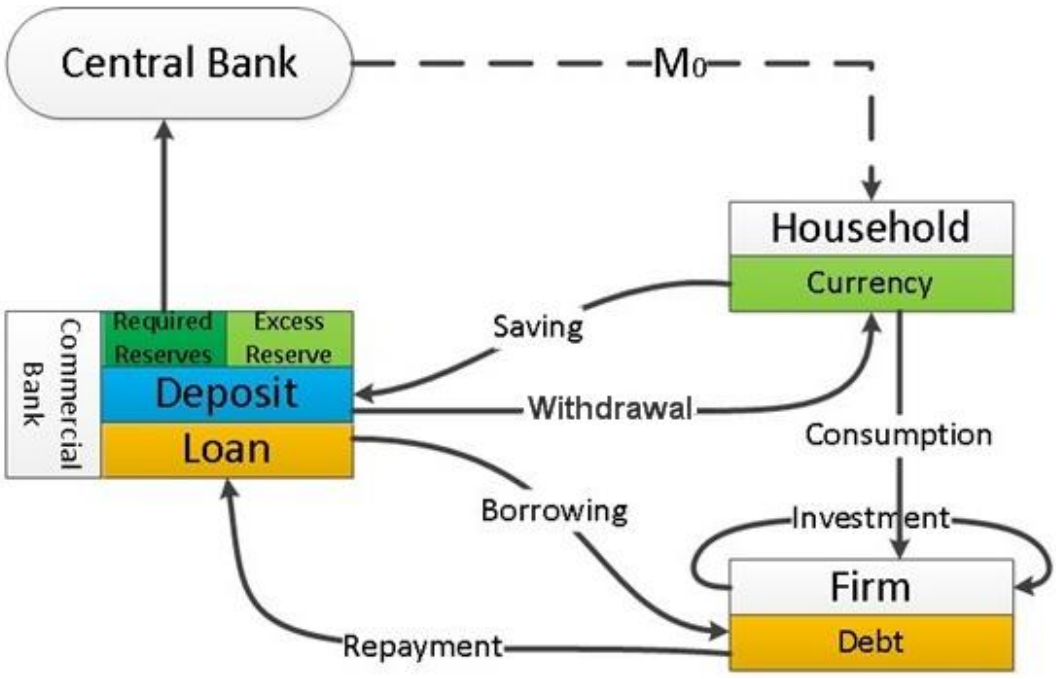

Figure 1: A schematic illustration of money creation process

The Model and Dynamic Equations

To describe mathematically the process mentioned above, we should integrate three stock variables: amount of currency $C(t)$, deposits $D(t)$, and outstanding loans $L(t)$. We also introduce four flow variables: saving flow $S(t)$,withdrawal $W(t)$, borrowing of firms $I(t)$, and repayment $R(t)$, each of which has a tight linkage with corresponding stock variable, specifying the behavior ofcorresponding sector.

At time t, households save money to commercial banks. The saving flow $S(t)$ is a portion(c_1) of the currencies $C(t)$ held by them, which is given by

$$S(t) = c_1 \cdot C(t). \qquad (5)$$

Meanwhile, households may have withdrawals when they need money. The withdrawal flow $W(t)$ is a portion (c_2) of the deposits $D(t)$, which has the following form

$$W(t) = c_2 \cdot D(t). \qquad (6)$$

When commercial banks face a demand for loans from firms, they will draw on excess reserves to make loans. The loans to firms $I(t)$ is a ratio (c_3) of the excess reserves $ER(t)$, which takes the form of

$$I(t) = c_3 \cdot ER(t). \qquad (7)$$

The excess reserves $ER(t)$ is the remaining of the deposits $D(t)$ after delivering the required reserves $r \cdot D(t)$ and subtracting total outstanding loans $L(t)$, i.e.,

$$ER(t) = D(t) - r \cdot D(t) - L(t), \qquad (8)$$

where r is the required reserves ratio. When firms repay the loans to commercial banks, the repayments $R(t)$ is assumed to be a portion (c_4) of the loans $L(t)$, which can be written as

$$R(t) = c_4 \cdot L(t). \qquad (9)$$

Since the saving is the inflow of the currencies, but the withdrawal is the outflow, the change in currencies at time *t*is the difference between them, which can be expressed as

$$\frac{dC}{dt} = W(t) - S(t) \ . \qquad (10)$$

The behavior of firms mainly determinesthe change in outstanding loans, which is the difference between borrowing and repayments:

$$\frac{dL}{dt} = I(t) - R(t) \ . \qquad (11)$$

In contrast to these two variables, the change in deposits stems from both households and firms. The saving and withdrawal contribute to deposits in the opposite direction as to the currencies.On the other side, the loans turn out deposits of firms and repayments annihilate them. Thus the difference between borrowing and repayments also makes up additional part of the change in deposits. So we have

$$\frac{dD}{dt} = S(t) - W(t) + I(t) - R(t). \qquad (12)$$

The above model presents the behavior of all sectors involved in the money creation system. From the model developed above, we can recognize that each sector plays a different role. These sectors exert influences on the money supply all the time.We should take a dynamic and systemic perspective to analyze it.

Integrating (5)-(12)results in the following system of equations:

$$\begin{cases} \dfrac{dC}{dt} = c_2 D(t) - c_1 C(t) \\ \dfrac{dD}{dt} = c_1 C(t) + [c_3(1-r) - c_2]D(t) - (c_3 + c_4)L(t) \\ \dfrac{dL}{dt} = (1-r)c_3 D(t) - (c_3 + c_4)L(t) \end{cases} \quad (13)$$

Each variable is changing. The change of a stock variable causes the change of a flow variable, and vice versa. They are all interconnected by the interactions of the sectors.

The Equilibrium Solution

Starting from the initial condition of $C(0) = M_0$, the system will eventually move to an equilibrium state, where there is no more change in any stock variable over time, that is

$$\begin{cases} \dfrac{dC}{dt} = c_2 D(t) - c_1 C(t) = 0 \\ \dfrac{dD}{dt} = c_1 C(t) + [c_3(1-r) - c_2]D(t) - (c_3 + c_4)L(t) = 0 \\ \dfrac{dL}{dt} = (1-r)c_3 D(t) - (c_3 + c_4)L(t) = 0 \end{cases} \quad (14)$$

The set of equations confirms that when the system falls into a steady state, the quantity of loans must equal to the quantity of repayments, and the level of saving flow must equal to that of withdrawal.This is also an equilibrium state of the behaviors of all sectors.

Solving the equations in (14)yields the result:

$$\begin{cases} \dfrac{C(t)}{D(t)} = \dfrac{c_2}{c_1} \\ \dfrac{L(t)}{D(t)} = \dfrac{c_3(1-r)}{(c_3 + c_4)} \end{cases} \quad (15)$$

When the system comes to the equilibrium state, there is a proportional relationship between C, D and L. From (10) and (11), we have

$$M_0 = C(t) + D(t) - L(t). \quad (16)$$

Then from (15) and (16) we can obtain

$$\begin{cases} D = \dfrac{M_0}{1+\dfrac{c_2}{c_1}-\dfrac{c_3(1-r)}{(c_3+c_4)}} \\ C = \dfrac{c_2}{c_1}\dfrac{M_0}{1+\dfrac{c_2}{c_1}-\dfrac{c_3(1-r)}{(c_3+c_4)}} \\ L = \dfrac{c_3(1-r)}{(c_3+c_4)}\dfrac{M_0}{1+\dfrac{c_2}{c_1}-\dfrac{c_3(1-r)}{(c_3+c_4)}} \end{cases} \quad (17)$$

This set of results is capable of demonstrating the intrinsic interactions of all sectors involved in the money supply. Currencies, held by households, are also determined by commercial banks, firms, even the central bank. Deposits and total loans, held by commercial banks and firms respectively, are also determined byother three sectors.

Using a generalized definition, the money supply is

$$M = C + D = \frac{M_0}{1-\dfrac{c_1}{(c_1+c_2)}\times\dfrac{c_3(1-r)}{(c_3+c_4)}} \quad (18)$$

Suppose that households retain all currencies first without saving at all , c_1 will be zero,then the money supply just stays at the level of monetary base without any expansion.In an extreme situation like bank run, it is possible that households withdraw all deposits and save none. Then the money creation will stop. An increase in c_1 ,other things being equal, means a reduction of currency leakage, would turn more monetary base into the deposit and create more money .

This principle also suits the commercial banks. If banks make no loans, the money supply is also just the monetary base. When banks make more loans, that is, a rise in c_3 , other things being equal, the money supply will also be larger. So households' saving and commercial banks' lending can be regarded as playing a core role to some extent in the process of money creation.

Different from the movement mentioned above, when households withdraw more money for their demands, which means a rise in c_2 , currency leakage will be more, then money circulated in the creation process will decrease, which will reduce the total money supply. When firms have more repayments, the same story goes. It is a way of money annihilation. Both of them should be worth to note. They may not cease the process, but actually exert heavy influences on the money creation.

Conclusion

In this paper, we discussed a miniature economy consisting of four sectors: central bank, commercial banks, firms and households. By considering the behaviors and interplays of the four sectors, we presented a stock-flow interpretation of the process of money supply in a dynamic model within a continuous time framework. With the help of the model, we then illustrate on the mechanism of money creation process and the determinants of the money multiplier based on the behaviors of the economic sectors. Specifically, when households hold lesscurrency and save more to commercial banks, the money supply will be more, i.e. the money multiplier will be larger. When commercial

banks make more loans, a similar story repeats. More withdrawals will increase the currency leakage, thus reducing the money supply, or the money multiplier, while firms' repayments will do the same.

A new contribution should be noted that we innovatively include households' withdrawals and firms' repayments into the story of money supply, which are long neglected in the conventional theories.

References

1. Phillips, C. A. 1920. *Bank Credit*. New York :Macmillan.
2. Samuelson, P. 1997. *Economic:TheOirginal 1948 Edition*. Irwin:McGraw-Hill.
3. Cagan, P. 1958. "The Demand for Currency Relative to the Total Money Supply," *Journal of Political Economy*, 66(4):303-328.
4. Ahrensdorf, J. and S. Kanesathasan. 1960. "Variations in the Money Multiplier and Their Implications for Central Banking," *Staff Papers-International Monetary Fund*, 8(1):126-149.
5. Teigen, R. L. 1964. "Demand and Supply Functions for Money in the United States: Some Structural Estimates," *Econometrica*, 32(4): 476-509.
6. Friedman, M. and A. Schwartz.1963. *A Monetary History of the U.S.1867-1960*. Princeton, N.J :Princeton Univ. Press.
7. Cagan, P. 1965.*Deterninants and Effects of Changes in the Stock of Money,1875-1960*. New York:Columbia Univ. Press.
8. Jordan, J. 1969. "Elements of Money Stock Determination," *Federal Reserve Bank of St.Louis Review*, 1969(10):10-19.
9. Brunner, K. and H. Meltzer. 1964. " Some Further Investigations of Demand and Supply Functions for Money," *The Journal of Finance*, 19(2): 240-283.
10. Bourne, C. 1976. "The Determination of Jamaica Money Stock: 1961-1971," *Social and Economic Studies*, 25(4):367-385.
11. McClean, A. W. A. 1985. "A Further Comment On Conventional Money Multiplier," *Social and Economic Studies*, 34(3):259-264.
12. He, P., L. X .Huang, and R. Wright. 2005. "Money and Banking In Search Equilibrium," *International Economic Review*, 46(2): 637-670.
13. Berardi, M. 2007. " Beyond the Static Money Multiplier In Search of a Dynamic Theory of Money," in *Artificial Markets Modeling*, A. Consiglio, ed. Berlin, Heidelberg : Springer, pp. 3-16.