

Project Planning and Scheduling, the Critical Path Method Approach. BUI Power Project as a Case Study

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ABSTRACT: This paper is aimed at minimizing project time subject to the available resources using Critical Path Method (CPM) as a planning and scheduling tool and the visual display of activities to showing project execution flow and coordination between activities based on a defined algorithm using the Bui Power Project (BPP) as a case study. Data was gathered from both primary and secondary sources from a sample of 50 selected respondents consisting of consultants, engineers and other workers of the Bui Power Project using non probability sampling techniques. Analysis of data was network based and use was made of Lingo software and GenStat in solving linear programming problems and performing some statistical test respectively. Sensitivity analysis and improvement to optimality using various methods were also looked at to meeting an optimum value.

Keywords: Project planning, scheduling, Critical Path Method, Network, Heuristics, Spanning tree

INTRODUCTION

A project is a set of activities to be performed in a specific sequence to completion. The execution of projects involves the coordination of many activities. Careful planning is required to ensure that a project is completed on time and within budget.

Project planning deals with the interrelationships between, and the timing of the various activities that comprise a project. Detailed planning of a project entails identification of the activities that comprise the project, estimation of the duration of each activity, identification of precedence relations between activities (i.e., which ones need to precede others) and development of an organizational network of schedule that represents this information accurately with the aim of minimizing the total project time subject to the resource constraints. Such organizational network can be used to provide the following information:

- i. The minimum time to complete the project if all activities run on time.
- ii. The activities that is critical to ensure that the project is finished in the minimum time.
- iii. The earliest start time and the latest finish time for each activity, if the project is to finished in the minimum time; and
- iv. The amount of time by which each activity can be delayed without delaying the project as a whole.

Critical Path Method is one of several Operations Research techniques developed to assist in project

planning. It is a systematic procedure for detailed project planning and control. Critical Path Method is a widely used technique for analyzing and managing sequential tasks in large projects for which the Bui dam is of no exception. Based on calculating how long it takes to complete essential steps of a process and analyzing how those steps interrelate, CPM is a visual and mathematical algorithm that gives managers the ability to effectively plan, schedule, and evaluate their projects.

RESEARCH DESIGN AND METHODOLOGY

As a result of the multiplicity of the research questions and diversity in the types and sources of data required for answering some questions, it became very apparent in the study that the data would be both qualitative and quantitative in nature.

A quantitative research strategy involving the use of a survey was adopted in the first stage for answering most of the research questions. This was followed by an in-depth qualitative investigation of issues through questionnaire administration. The survey also helped in the identification of appropriate interviewees and a major factor that influenced the choice of the survey strategy was the large and diverse nature of the research population.

The set of activities that comprise the project and the precedence relationships between those activities constitute the secondary data. This type of data is required in the network construction.



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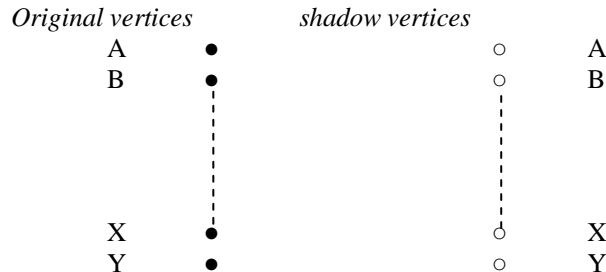


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ALGORITHM FOR ACTIVITY NETWORKS CONSTRUCTION

For simple problems, it is relatively easy to construct activity networks but, as the complete project becomes more complex, the need for a formal method of constructing activity networks increases, such an algorithm is summarized below:

STEP 1: produce a list of all the original vertices and a copy as shadow. With the help of precedence relationships, connect original nodes to shadow nodes using lines/arcs. As illustrated below; if activity Y precedes activity X, draw an arc from original vertex Y to shadow vertex X.



STEP 2: Identify and delete all vertices (original and shadows), which have no arcs incident to them.

STEP 3: Repeat steps 1 and 2 until all the vertices are considered.

TABLE 1: MAIN ACTIVITIES AND THEIR PREDECESSORS WITH ESTIMATED DURATIONS

ACTIVITIES	IMMEDIATE PREDECESSORS	DURATIONS/M ONTHS
1. Turnkey contract award [A]	-	0
2. Detail design and additional investigation [B]	A	12
3. Preparatory works - [C]	A	12
4. Site mobilization [D]	B	4
5. River diversion [E]	D	9
6. Main dam [F]	D	31
7. Saddle dams [G]	C,E	17
8. Power station [H]	C,F,G	55

Source: Bui Power Authority

Based on the defined algorithm, the project network of the BPP is:

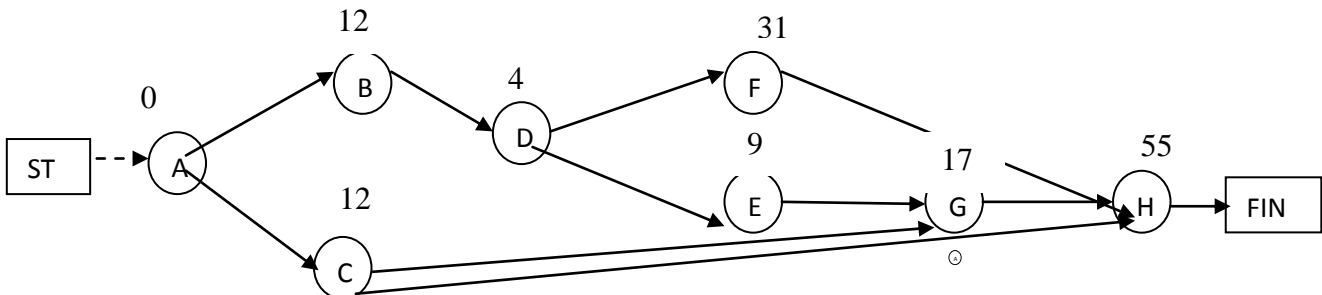


Figure 1: Activity - On - Node Network of the BPP based on Stated Algorithm

LINEAR PROGRAMMING PROBLEM MODEL FOR A PROJECT NETWORK

Define

$x_j =$ The time that the event corresponding to node j occurs

For each connected nodes (i, j) , we know that before node j occurs, node i have occurred. This implies that for each connected nodes (i, j) in the project network;

$$x_j - x_i \geq D_{i,j}$$

Let F be the node that represents completion of the

project. Since the goal is to minimize the project completion time, the objective function is set as;

$$Z = x_F - x_1$$

And the non negativity constraints set as $x_i \geq 0$ for all i 's

ASSUMPTIONS

- i. There exist linearity between node i and node j in a project network.
- ii. (7) Activities durations are considered discrete in form.

RELATIONAL PROPERTIES TEST OF CONNECTED NODES

Relationships between elements of sets occur in many contexts and including nodes of a connected network of the BPP.

Let R be the set of all ordered pairs of connected nodes of the BPP. i.e.,
 $R = \{ (1,2) (2,3) (2,4) (3,5) (4,8) (4,9) (5,6) (5,7) (6,8) (7,9) (8,9) (9,10) \}$

- i. If there exist $\{(a, b) (b, c)\}$, does the pair (a, c) exist in R?
- ii. If there exist an element a in R, does the pair (a, a) exist in R?
- iii. If there exist the pair (a, b) in R, does the pair (b, a) also exist in R?

THE CRITICAL PATH

TABLE 4.4.1: THE PATHS AND PATH LENGTHS THROUGH THE BPP

PATH	LENGTH/MONTHS
START → A → B → D → F → H → FINISH	$0+0+12+4+31+55 = 102$
START → A → B → D → E → G → H → FINISH	$0+0+12+4+9+17+55 = 97$
START → A → C → G → H → FINISH	$0+0+12+17+55 = 84$
START → A → C → H → FINISH	$0+0+12+55 = 67$

The longest path through the Bui Power Project network is 102 months. No duration will be longer than this particular path. This is the longest path through the project network. The activities on this path can be performed sequentially without interruption. Therefore,

the time required to reach the FINISH node equals the length of this path. Furthermore, all the shorter paths will reach the FINISH node no later than this. The conclusion here is:
 For the Bui Power Project, the critical path is:

$$START \rightarrow A \rightarrow B \rightarrow D \rightarrow F \rightarrow H \rightarrow FINISH$$

The activities on this Critical Path are the critical bottleneck activities where any delays in their completion must be avoided to prevent delaying the project completion. Management of the Bui Project should focus attention on keeping these particular activities on schedule in striving to keep the overall project on schedule.

OPTIMUM PRECEDENCE RELATIONS USING MINIMUM CONNECTOR CONCEPT

Minimum connector concept is applied where all nodes are connected without a cycle and at a minimum arc length using the prim's algorithm. Suppose that each arc (i, j) in a network has a length associated with it and that arc (i, j) represents a way of connecting node i to j. In such application, we want to determine the set of arcs in a network that connect all nodes such that the sum of the length of the arcs is minimized.

DURATION MATRIX OF ALL CONNECTED NODES OF THE NETWORK

	1	2	3	4	5	6	7	8	9	10	
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
	START	A	B	C	D	E	F	G	H	FINISH	
START	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	→ 1
A	0	∞	0	0	∞	∞	∞	∞	∞	∞	→ 2
B	∞	0	∞	∞	12	∞	∞	∞	∞	∞	→ 3
C	∞	0	∞	∞	∞	∞	∞	12	12	∞	→ 4
D	∞	∞	12	∞	∞	4	4	∞	∞	∞	→ 5
E	∞	∞	∞	∞	∞	∞	∞	9	∞	∞	→ 6
F	∞	∞	∞	4	∞	∞	∞	∞	31	∞	→ 7
G	∞	∞	∞	12	∞	9	∞	∞	17	∞	→ 8
H	∞	∞	∞	12	∞	∞	31	17	∞	55	→ 9
FINISH	∞	∞	∞	∞	∞	∞	∞	∞	55	∞	→ 10

∞ represents no direct connections between the nodes

Assuming both cases of an un-directed and connected network with given time/month associated with each link and with the application of prim's algorithm to build up a spanning tree step by step, connecting edges into existing solution.

Applying algorithm to the time matrix,

1. Choose a starting vertex say COLUMN LABEL 1(START), delete the START row. Look for the smallest entry in the START column. This value is 0 at ROW A. Connect A node to the START node and put the edge START — A into the solution.
2. Delete row A and look for the smallest entry in the COLUMNS START and A. This value is 0 in ROW B (breaking

ties arbitrarily). Connect B to A and put the edge — A in the solution to form START — A — B.

3. Delete ROW B and look for the smallest entry in the COLUMNS START, A and B. This value is 12 at ROW D and put edge B — in the solution. Continue the process until all nodes are connected their minimum times. Break ties arbitrarily if they exist in the process.

This algorithm generates a spanning tree in which all nodes of the network are connected with a minimum arc length of 84 months. This indicates that the Bui Power Project can be completed within a period of 84 months with the precedence relation:

ACTIVITY DESCRIPTION	PRECEDENCE RELATION
START	-
A	START
B	A
C	B
D	B
E	D
F	D
G	E
H	C
FINISH	H

HEURISTIC WORKERS SCHEDULING TECHNIQUES

In the Critical Path Method, the determination of minimum time and Critical Path is possible. This schedule is however, limited in the sense that consideration is not made for workers available to undertake the activities, or indeed to how many workers would be needed for each activity.

SCHEDULING OBJECTIVE

Complete the project as soon as possible with the available number of workers

SCHEDULING RULES

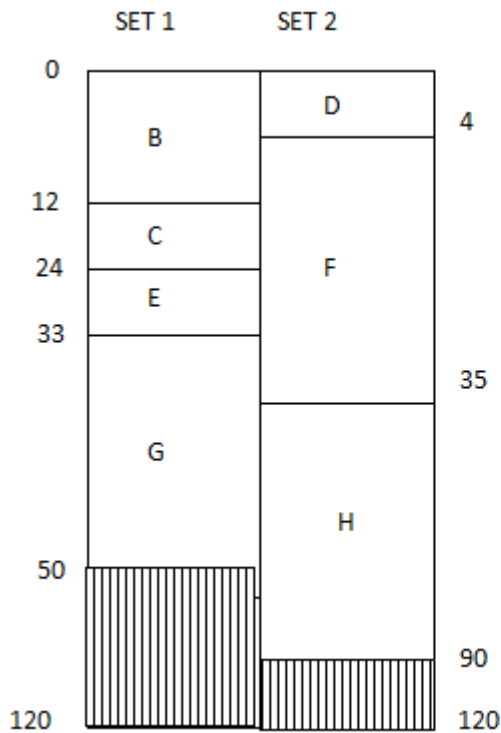
- i. Each activity requires some SET of workers
- ii. No SET of workers may be idle if there is an activity that can be started.

- iii. Once a SET starts an activity, it must be continued by that SET until it is completed.
- iv. At any stage, when a worker/SET becomes free, consider all the activities which have not yet been started but which can now be started. Assign to the workers/SET the most CRITICAL one of these (i.e. the one who's LATEST START time is the smallest). If there are no activities which can be started at this stage you may have to wait until the workers can be assigned a job.

Using this as a basis for decision, the model is obtained as:

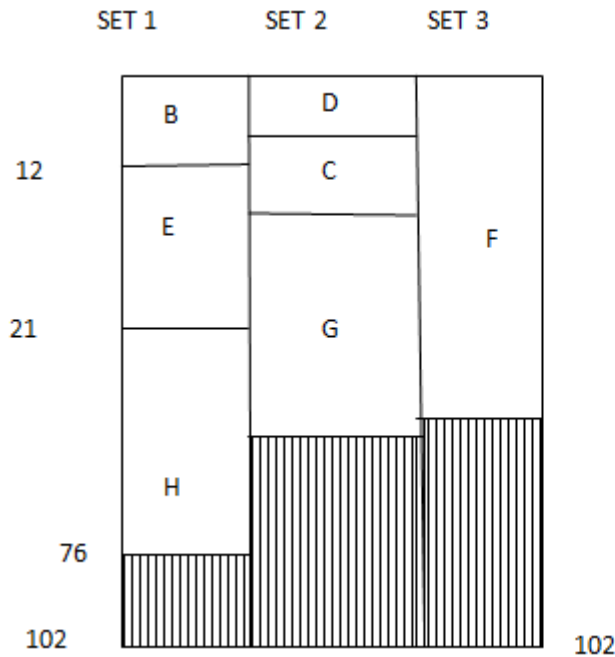
WORKERS SCHEDULING MODELS

I. TWO SETS MODEL



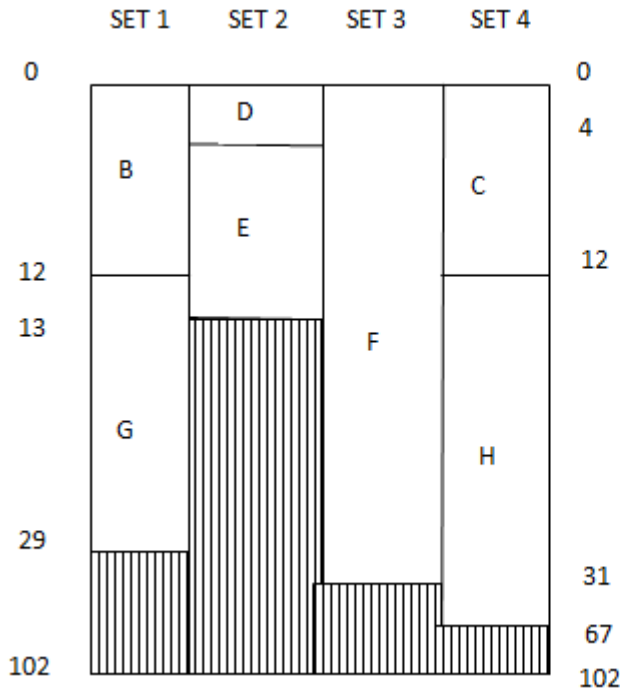
This schedule produces a minimum time of 90 months to complete the project.

II. THREE SETS MODEL



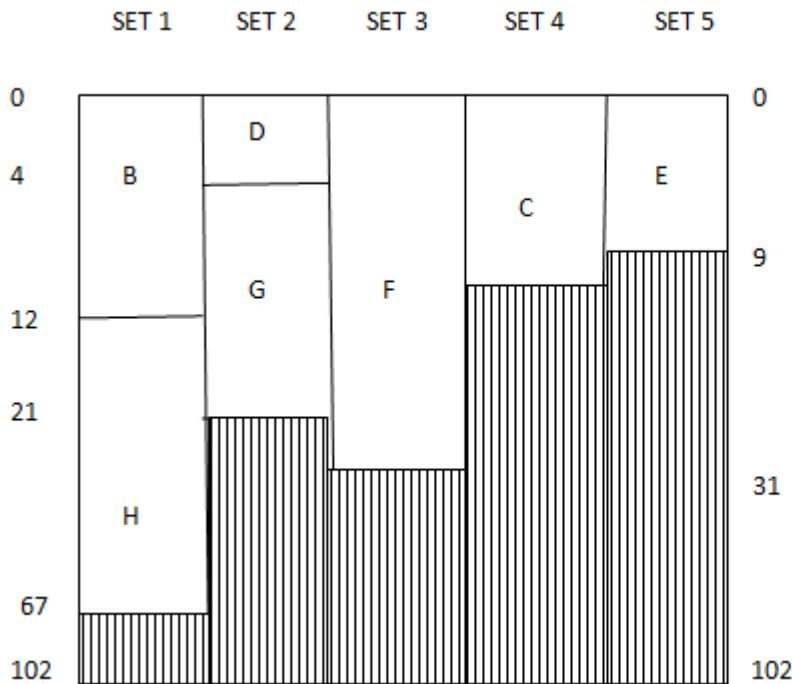
This schedule produces a minimum time of 76 months to complete the project if no delays occur.

III. FOUR SETS MODEL



This schedule produces a minimum time of 67 months to complete the project if no delays occur.

IV. FIVE SETS MODEL



This schedule produces a minimum time of 67 months to complete the project if no delays occur.

CONCLUSION

An optimum solution found if groupings are done according to the number of CRITICAL ACTIVITIES available.

For each grouping:

$$\text{Number of workers per an ACTIVITY} = \frac{N \times T_i}{T_m}$$

Where N = Total number of

available workers

T_i = Duration/ time for activity i.

T_m = Minimum time required to complete the project.

STATISTICAL ANALYSIS OF QUESTIONNAIRE DATA

In this regards, special attention is paid to the use of CPM, it's awareness, concerns of CPM

implementation, other planning techniques used, level of management where CPM is used, areas of construction where CPM is used, etc.

The questionnaire responses are tabulated reflecting number of responses in percentages with respect to each category of workers.

HYPOTHESIS I

H₀: Application of CPM in all areas of project management is the same.

H₁: Application of CPM differs from sector to sector in project management.

GENSTAT OUTPUT

CHISQUARE[PRINT=test,prob,fitted,tchisquare; METHOD=pearson] table

Pearson chi-square value is 0.90 with 6 df.

Probability level (under null hypothesis) p = 0.989

HYPOTHESIS II

H₀: Knowledge of computer software availability among workers of the BPP is the same.

H₁: Knowledge regarding computer software differs among all worker categories.

This hypothesis is tested using the CHOCHRAN Q TEST at 5% level of significance.

TEST STATISTIC

$$Q = \frac{(k - 1)[k\sum s_i^2 - (\sum L)^2]}{k(\sum L) - \sum L^2}$$

$$= \frac{25494}{233} = 109.42$$

k is the number of sub samples. In this case, workers categories
∑s_i is the sum of the favorable (effective responses to the available softwares)
L is the total number of favorable response for each available software

HYPOTHESIS III

H₀: CPM is used as a planning tool in the Bui Power Project.

H₁: CPM is not used in the Bui Power Project as a planning tool

TEST STATISTIC

$$\chi^2 = \sum \sum \frac{[n_{ij} - E(n_{ij})]^2}{E(n_{ij})}$$

n_{ij} Represents number of counts in each cell

E(n_{ij}) Represents expected cell count

$$\chi^2 = \frac{[2-1.6]^2}{1.6} + \frac{[14-14.4]^2}{14.4} + \frac{[0-0.4]^2}{0.4} + \frac{[4-3.6]^2}{3.6}$$

$$= 5.56$$

But $\chi^2_{(0.05,1)} = 3.84146$

Thus $\chi^2_{cal} > \chi^2_{(0.05,1)}$

CONCLUSION AND RECOMMENDATION

CPM is one of several operations research techniques developed in the late 1950s to assist project managers in planning, scheduling and controlling their projects. It is a visual and mathematical algorithm that gives managers the ability to effectively plan, schedule and evaluate their projects and largely viewed as the representation of a project plan by a schematic diagram or network that depicts the sequence and interrelation of all the component parts of a project, and the logical analysis and manipulation of this network in determining the best overall program of operation or the best project completion time.

The study looks at the possibility of application of CPM as a planning tool in the Bui Project which is aimed at minimizing the project completion time subject to the available resources. It is also aimed at producing a project network subject to a developed algorithm to visually display the relationships between activities and their predecessors using Activity-on-Node network, where each activity is represented by a node.

The network provides the order in which certain activities must be performed and the durations of each activity which are deterministic in nature. These information's help in addressing:

- i. The total time required to complete the project if no delays occur.
- ii. The times the individual activities need to start and finish (at the earliest and at the latest) if no delays occur
- iii. The critical bottleneck activities where any delays must be avoided to prevent delaying the project completion time.
- iv. The amount of delays that can be tolerated for a particular activity without delaying the project completion.

The study employs both primary and secondary data. The set of activities that comprise the BPP and their estimated durations constituted the secondary data. This data is used to construct the project network. The primary data is obtained from a sample of 50 respondents through questionnaire administration using a non probability sampling techniques. Statistical analysis including hypothesis testing using the Chockran Q test, the chi-square contingency test and the Genstat are performed on sampled data and tested at 5% level of significance and the use of the P values in making decisions of either rejecting or accepting the null hypothesis.

On the assumption of linearity between the time that an activity at a particular node occurs and the estimated durations, objective function and other constraints of a linear programming problem of the project network is solved via lingo coding.

Introducing uncertainty aspect into the deterministic nature of CPM moves analysis into the realm of Project Evaluation and Review Technique (PERT) making CPM and PERT closely related Operations Research techniques. This PERT statistical approach to project planning leads to a probability (0.8413) that the given scheduled event occurrence time will be met, without having to expedite the project. This signals a green light to go with the project plan.

Moder et al (1970), indicate, probability values of greater than say 0.75 signals a green light to go with the project plan, values between 0.25 – 0.75 signals a yellow light which anticipates some re-planning and probability less than 0.25 signals danger and requires immediate halting and re-planning to achieve a better chance of meeting the required schedule.

The study revealed that if no delays occur, a minimum of 102 months is required to completely execute the project. By the means of minimum connector principle and other heuristic measures, it is possible to complete the project either within 84 months or 67 months. These estimates produce probability estimated values of about 0.84 and 0.96 respectively. Furthermore, activities B, D, F and H must not be allowed to delay even for a day if indeed the project is aimed at meeting the minimum time.

Further findings revealed that CPM is not specifically applied in the BPP and time estimation is done through Microsoft office Project. Again, knowledge regarding availability of computer software that aid project management and planning is skewed towards those with the technical knowledge. Other management problems such as monitoring, controlling, updating to mention but a few are met through MS project. These findings have emanated from a section of the workers whilst research hypothesis have been supported with this data. Generalization however, is theoretical justifiable since the principle of sampling is used and sample seems representative.

Base on research findings, it is recommended that:

- i. CPM should be applied in all stages of project management. This will allow project to be completed earlier than the project minimum time.
- ii. Every worker should be abreast with the principles and application of CPM.
- iii. Incorporate spanning tree principle into the CPM concept.
- iv. Further research done on the possibility of refining CPM to include ways of scheduling workers to all activities for optimum resource allocations.

It is our belief that there is a bright future for CPM application in dam construction if more efficient

heuristics are developed via computer software

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