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# Modeling Domestic Violence and Predicting its Growth using Differential Equations. A Case Study of Women and Children in Tamale

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**ABSTRACT:** The paper focused on analytic and numerical modeling of Domestic Violence. In the case of the analytic modeling, this paper discusses a simple continuous model for the spread of Domestic Violence, using Ordinary Differential Equations. A mathematical model is inspired from the spread of Domestic Violence in Tamale Metropolitan in which the interaction of the widespread is likely to be minimized. A modeling technique Abusive, Susceptible and Violence Victims (ASV), similar to the Susceptible, Infectious and Recovered (SIR) model in Epidemics, is used for formulating the spread of Domestic Violence as a system of Differential Equations. Hence the population of three distinct classes- the Abusive, Susceptible/Unreported Victims, and the Reported Victims, are considered in the model. The system of Differential Equations is analyzed by linearization of nonlinear systems and non-dimensionlization, and to predict the behavior of the spread of the Domestic Violence. Finally, in the case of the numerical analysis, a general model for the population of Domestic Violence Victims is constructed. The present model shows that the given data is reasonably Logistic. Moreover, this model shows that the population of Domestic Violence Victims is limited. A projected limiting number is given by this model. Some typical mathematical models are introduced such as Exponential model and logistic model. The solutions of those models are analyzed.

**Keywords**: Domestic Violence, Ordinary Differential Equations, linearization, ASV.

## 1.0 INTRODUCTION

The provisional results of the 2010 Population and Housing Census shows that total population of Ghana is 24,233,431 (11,801,661 males and 12,421,770 females). The males form 48.7 percent of the population and females constitute 51.7 percent. The total population in Northern Region is 2,468,557 (1,210,702 males and 1,257,855 females, Ghana Statistical Service, 2010). Tamale is a bustling Regional Capital of the Northern Region about 400 miles north of the Atlantic Coast in West Africa. The geographical area of Tamale is about 750 kilometer square (289.58 square miles). The economy of the area is predominantly Agricultural. Most people in Tamale are muslems and they practice the polygamous system of marriage and also have large family size. The dominant tribe is the Dagombas but there are other tribes like Gonja, Mamaprusi, Konkomba, Hausa, Dagarti, and Grusi. Globally, in 90 percent of the cases of Domestic Violence, the victim is a woman and 90 percent of the time the abuser is a man. It is also estimated that one out of four women will experience intimate partner violence at some time in their life (Perttu-Rautava, 2002). Violence against women and children continue to be a global epidemic that kills, tortures, and maims – physically, psychologically, sexually and economically. It is one of the most pervasive of human rights violations, denying women and children equality, security, dignity, self-worth, and their right to enjoy fundamental freedoms. Violence against women and children is present in every country, cutting across boundaries of culture, class, education, income, ethnicity and age. Even though most societies proscribe violence against women, the reality is that violations against women's human rights are often sanctioned under the garb of cultural practices and norms, or through misinterpretation of religious tenets (Manuh, 2002).

Domestic violence has been outlined by Amnesty International as most violent attacks on an individual or group of people or women. It involves physical, sexual and psychological violence in the family including battering, sexual abuse of female children in the household, dowry related violence, marital rape, female genital mutilation and other traditional practices harmful to any member of the household. Domestic violence may be exhibited by any member of the household.

All over the World, governments and non-governmental and international organizations are

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trying to collaborate to come out with appropriate ways to handle these burning issues. On the international front, (Avoke et al, 1999) cited the United Nations declaration (1998) which enjoined member states to protect the rights of citizens, particularly the vulnerable groups such as women, children, the disabled and disadvantaged. The repercussions of these violent acts and how spouses and their children suffer thereafter appears to be taken for granted. Women all over the World agitated and fought for human dignity, more especially, the dignity of women and Children. Significant among these moves was the Beijing Platform for Action (BPFA, 1998). In Ghana, a Ministry is established to focus on the interest of women and children. The philosophies behind its establishment were borne out of the idea that, women and children are vulnerable and are prone to domestic violence. Another important unit was also set up in the Police Service that is Women and Juvenile Unit (WAJU) to handle cases concerning women and juveniles in the society and it is now called Domestic Violence Victim Support Unit (DOVVSU). International Federation of Women lawyers (FIDA) is one of the organizations in Ghana which plays advocacy roles in combating all kinds of violence against children and women in society.

Ironically many people in Ghana generally tend to see domestic violence as part of everyday experiences. (Cusack et al, 1999) remarked of a queen mother who was reported to have commented as follows "I hear every day that somebody has beaten the wife but women do not come to report the men. I also keep quite in my house" Such comments underpin the mini-subculture of abuse and violence that exist in many homes, and their subsequent effect on children growing up in those homes. In many communities in Ghana, it is often the case that when defilement, assault, sexual harassment, rape, and battering are reported, little attention or regard is giving to them thereby, reinforcing the perpetuation of violence in many homes. On February 2007, Ghana's Parliament passed the much-awaited Domestic Violence Bill (DVB), which had been laid before it in 2003 and had been the subject of heated debate. The process leading to the passage of the law involved not only the introduction of new legislation, but also confronting a social system that tolerates various forms of violence against women and children, especially in the context of gender relations and in the domestic sphere. It is against this background that this study is being conducted to ascertain the current state of domestic violence and its growth.

## 2.0 MATERIALS AND METHODS

### 2.1 DATA

In order to evaluate the descriptive adequacy of the model, we obtained data sets on Domestic Violence against women and children. The data used for the model was reported Domestic Violence cases at DOVVSU, Tamale police station. The data used was yearly reported cases from 1999-2011.

### 2.2 MODEL DEVELOPMENT

#### 2.2.1 Malthusian Model

According to the Malthusian model (Unlimited population growth) an elementary model of population growth is based on the assumption that; the rate of growth of the population is proportional to the size of the population. This implies that the rate of change of a population depend only on the size of the population and nothing else. Exponential functions come into play in situations in which the rate at which some quantity grows or decays (i.e., increases or decreases over time) is proportional to the quantity present .The quantities evolving from the assumption are as follows;

$t \rightarrow time$ (Independent variable)

$N \rightarrow Total\ Population\ size\ in\ Tamale\ metropolitan$

$V \rightarrow Population\ of\ Reported\ Domestic\ Violence\ Cases$ (Dependent variable) and

$\alpha \rightarrow Proportional\ constant\ (parameter)$, between the rate of growth of the population $N$ and the size of the population of Violence Victims $V$. α tells us how fast the population of victims of Domestic Violence is changing at any given population level. It could be positive or negative. If α is positive, it means the population of victims is increasing and if α is negative, it means population of victims is decreasing.

Based on the assumption the rate of growth of Population of Domestic Violence Victims (V) is the derivative $\frac{dV}{dt}$, implies

$$\frac{dV}{dt} = \alpha NV \qquad (1)$$

By separation of variables, the solution to the differential equation of first order $\frac{dV}{dt} = \alpha NV$, subject to $V = V_o$, at $t = 0$ is given by;

$$V(t) = V_0 e^{\alpha Nt}$$

That is, if we know that the number of Domestic Violence Victims, at time $t = 0$, is $V_o$ and that the rate of change is $\frac{dV}{dt} = \alpha NV$, then we will be able to find an expression for the number of Domestic Violence Victims present at time, (t) as

$$V(t) = V_0 e^{\alpha N t} \qquad (2)$$

And this model is called the exponential model. This model presents exponential growth without limit. However, in our real world, this case does not happen, because we need to consider environmental factors, food, drugs, law, education etc. We choose this model in order to start from the simplest model and based on it try to involve some factors step by step coming close to describing the actual situation and approach the goal.

### 2.2.2 The Logistic Growth Model

We will divide the population into two groups:

- Susceptible/Abusive individuals at time (t), $S(t)$
- Domestic Violence Victim's at time (t), $V(t)$
- Total population size, $N$

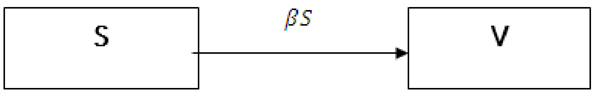


Figure (1)

### 2.2.3 Assumptions of the Model

- Population size is large and constant,
  $S(t) + V(t) = N \qquad (3)$
- No birth, death, immigration or emigration
- No recovery
- Homogeneous mixing
- Violence spread rate is proportional to the number of Domestic Violence Victims, i.e. $\beta V$

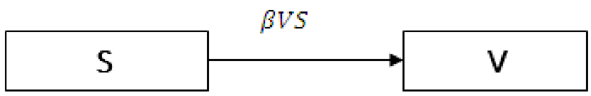


Figure (2)

A pair of ordinary differential equations will describes this model:

$$\frac{dS}{dt} = -\beta V(t) S(t) \qquad (4)$$

$$\frac{dV}{dt} = \beta V(t) S(t) \qquad (5)$$

But $N = S(t) + V(t)$, so this is equivalent $S(t) = N - V(t)$ and substituting into equation (5) gives

$$\frac{dV}{dt} = \beta V(t)[N - V(t)]$$

The differential equation is known as the **Logistic Growth Model.**

We will have a nonlinear ODE,

$$\frac{dV}{dt} = \beta V(t)[N - V(t)] \qquad (6)$$

And finally in the form

$$V(t)' = \beta V(t)\left(1 - \frac{V(t)}{N}\right) \qquad (7)$$

First, the derivative will be zero at $V(t) = N$. Also; these are in fact solutions to the differential equation. These two values are called equilibrium solutions. If we start with a population of zero, there is no growth and the population stays at zero. If we start with a population in the range $0 < V(t) < N$, then from the differential equation we know that $\frac{dV}{dt} > 0$ and hence $V(t)$ is increasing. If we start at $V(t) = N$, the population stays at this level. Similarly, if we start with $V > N$, then $\frac{dV}{dt} < 0$ and hence $V(t)$ is decreasing. Using the analysis we construct the following phase line diagram shown in the figure below.

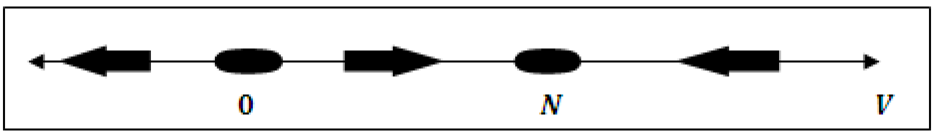


Figure (3)

From the above phase line diagram, we see that solutions tend toward the equilibrium at N and hence the solution $V(t) = N$ is stable while the equilibrium at 0 is unstable. According to this model, if the population of victims of Domestic Violence is above 0, it will go to the carrying capacity N eventually.

Equation (6) is separable so we will divide,

$$\frac{1}{V(t)[N - V(t)]} \frac{dV}{dt} = \beta$$

And integrate,

$$\int \frac{1}{V(t)[N - V(t)]} \frac{dV}{dt} dt = \int \beta dt$$

Let $u = V(t)$, implies $du = dV$

Therefore,

$$V(t) = \frac{NCe^{\beta t}}{1+Ce^{\beta t}} \qquad (8)$$

At t=0

$$C = \frac{V(0)}{N-V(0)}$$

From equation (8), we have

$$V(t) = \frac{NV(0)}{V(0)+[N-V(0)]e^{-\beta t}} \qquad (9)$$

Analyzing the solution above, we see that as time increases, the size of the population of victims reaches a finite limit. Mathematically, as $t \to +\infty$, $V \to N$ and almost all women and children becomes victims of Domestic Violence.

**2.2.4 The Violence Epidemic Model (ASV)**

The ASV model just as the SIR Model in epidemiology will be used for the analysis. The victims will be mainly restricted to Tamale Metropolis. Therefore, we assume that the committing of Domestic Violence occurs mostly within a high-risk population group, such as men, women and children. Besides, Domestic Violence Victims, who are at the end-stage of the Domestic Violence infection, are strictly isolated from Abusive and susceptible population; therefore, they no longer cause any form of Violence. To simplify a real problem, we assume;

**Assumptions:**

- ❖ A particular population, which is reasonably restricted, is at high-risk to Domestic Violence by contact only.
- ❖ The population is uniformly mixed, so the probability of a person been a victim of Domestic Violence equally exists to every single individual within the community.
- ❖ Once victims (Women and Children) are classified into Domestic Violence Victims, they are no longer engaged in the spread of the violence.
- ❖ There are no subtractions of the population except for Violence-induced death.

**2.2.5 Formulated Mathematical Model**

For the Domestic Violence victim modeling, we will consider a group of susceptible individuals; therefore a number of susceptible individuals (Women and Children) are introduced into a larger susceptible population. Besides, the infected individuals develop themselves into a group of Domestic Violence Victims, the end-stage of the violence. The population of each group will change respect to time while the violence progresses. Therefore, we will think of the change of the population as a mass balance idea,

**Rate of change in Domestic Violence population** = **population growth – population loss**.

Then, the terms for the population growth and population loss can be defined by the interaction of each population group in the transmission of violence. To mathematically translate, we will introduce a system of ordinary differential equations. Let's declare variables before creating ODEs.

**Definition of Variables and Parameters**

- ❖ $N$ - Total population of a community
- ❖ $\mu$ - Recruitment rate to a population
- ❖ $\beta$ - Conversion rate (from Susceptible to Victim)
- ❖ $S(t)$ – Number of Susceptible individuals/unreported domestic violence victims at time (t)
- ❖ $V(t)$ – Number of Reported Domestic Violence Victim's at time (t)
- ❖ $A(t)$ – Number of Abusive individuals at time (t)
- ❖ $\alpha$ – Susceptible/Unreported Victims rate
- ❖ $\delta$ - Violence-induced death rate

The number of each population changes in time, so it can be expressed as a function of time, and the total population, $N(t)$, consists of three sub-classes, $A(t)$, $S(t)$, and $V(t)$. First, let's consider the rate of change in the Susceptible individuals, $S(t)$, which represents all women and children in the population who are exposed to Domestic Violence. Since there is a population growth at a rate, $\mu$, new incomers will immediately belong to the Abusive group, $A(t)$. Then, these individuals may be Susceptible by violent contact with in the Abusive group. Note that the infection occurs at a rate proportional to the number of the Susceptible and Abusive; that is $\alpha AS$, where a universal parameter, $\alpha$, represents all infective factors according to our assumptions. Thus, once individuals are infected, there will be the subtraction of population at the rate, $\alpha AS$, from $A(t)$ . Along with the population growth at the rate, $\mu$, we finally have the equation $\frac{dA}{dt} = \mu - \alpha A(t)S(t)$. Simultaneously, the infection will also invoke addition to $S(t)$ at the same rate, $\alpha AS$. In the same manner, the development of Susceptible individuals, $S(t)$, into Domestic Violence Victim at a certain rate will result the subtraction of population from $S(t)$ and the addition of population to the Domestic Violence Victim, $V(t)$. Let $\beta$ be the conversion rate from the Susceptible to Domestic Violence Victim, then the subtraction and the addition from $S(t)$ to $V(t)$ can be expressed as $\beta S$. Finally, Domestic Violence Victim may die at a rate $\delta$, which is the violence-induced death rate. As a result, a simple model can be formed as a system of deferential equations,

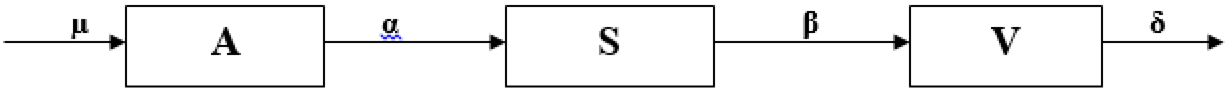


Figure (4)

$$\frac{dA}{dt} = \mu - \alpha A(t)S(t) \qquad (10)$$

$$\frac{dS}{dt} = \alpha A(t)S(t) - \beta S(t) \qquad (11)$$

$$\frac{dV}{dt} = \beta S(t) - \delta V(t) \qquad (12)$$

We will note that the system of the differential equations is a nonlinear 3-dimensional system. Also, we will note that equation (10) is an inhomogeneous equation since it has a recruitment rate $\mu$ per unit time to the Abusive class. As a result, the total population $N(t) = A(t) + S(t) + V(t)$ will not be constant.

The $\mu$ term is much like a "forcing term" which often arises from an external force in a physical application. The reason that we need to mention the $\mu$ term will be explained later when we analyze the system.

## 3.0 RESULTS AND DISCUSSION

### 3.1 ANALYSIS OF THE EPIDEMIC (ASV) MODEL

Unfortunately, we cannot explicitly solve the system of the differential equations, (10), (11) and (12). Instead, we can import an analytic approach to study the behavior of the infection. In the system, (10), (11) and (12), $\frac{dA}{dt}$ and $\frac{dS}{dt}$ are independent on Domestic Violence Victims, $V$. In other words, Domestic Violence Victims no longer affect the primary infection, so we will consider $\frac{dA}{dt}$ and $\frac{dS}{dt}$ only. Then, we have an equivalent 2-dimensional system for our model,

$$\frac{dA}{dt} = \mu - \alpha AS \qquad (13)$$

$$\frac{dS}{dt} = \alpha AS - \beta S \qquad (14)$$

$$From\ \frac{dA}{dt} = \mu - \alpha AS$$

But $\frac{dA}{dt} = 0$

$$0 = \mu - \alpha AS$$

$$\alpha AS = \mu$$

$$A = \frac{\mu}{\alpha S} \qquad (15)$$

$$Also\ from,\ \frac{dS}{dt} = \alpha AS - \beta S$$

$$\frac{dS}{dt} = 0$$

$$0 = \alpha AS - \beta S$$

$$\beta S = \alpha AS \implies A^* = \frac{\beta}{\alpha} \qquad (16)$$

Therefore, from equation (4.03)

$$\alpha AS = \mu$$

$$\alpha S\left(\frac{\beta}{\alpha}\right) = \mu$$

$$\beta S = \mu \Longrightarrow S^* = \frac{\mu}{\beta} \qquad (17)$$

Primarily, we're interested in equilibrium, if any, at which both $A$ and $S$ do not change; that is, neither increasing nor decreasing. For that, we need to find $A^*$ and $S^*$ such that both $\frac{dA}{dt} = 0$ and $\frac{dS}{dt} = 0$. To do that, we will look at the intersection of $A$ and $S$.

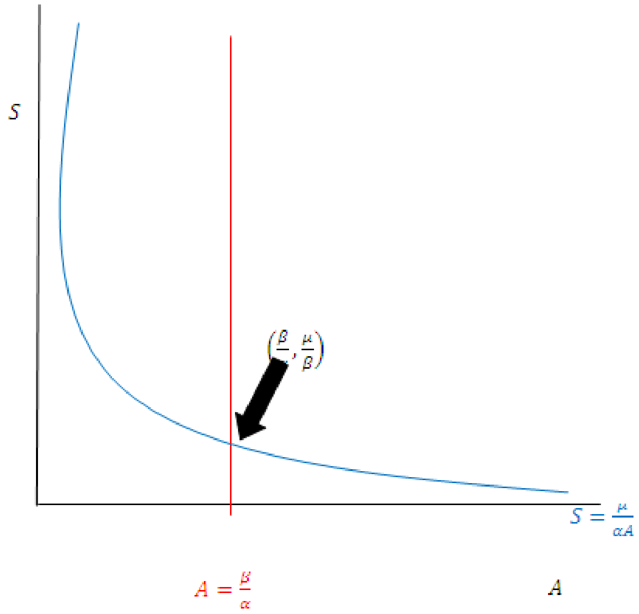


Figure (5)

We have one equilibrium point $E(A^*,S^*)=\left({}^{\beta}/_{\alpha},{}^{\mu}/_{\beta}\right)$ in the first quadrant. Thus, when $A^*={}^{\beta}/_{\alpha}$ and $S^*={}^{\mu}/_{\beta}$, the infection results a condition in which there is no change in the population of the Abusive individuals and the Susceptible and unreported victims of domestic violence. However, this is not quite enough information to predict the behavior of the violence. We need to know whether the equilibrium is stable or unstable. Since the system is nonlinear, we need to linearize the system at the equilibrium $E(A^*,S^*)$ to determine its stability by taking the *Jacobian* matrix, which is defined as,

$$J(A^*,S^*)=\begin{pmatrix}\frac{\partial f}{\partial A} & \frac{\partial f}{\partial S}\\ \frac{\partial g}{\partial A} & \frac{\partial g}{\partial S}\end{pmatrix}_{|(A^*,S^*)}$$

Where,

$\frac{dA}{dt}=f(A,S)$ and $\frac{dS}{dt}=g(A,S)$

Then, the linearization of the system at the equilibrium $E$ can be calculated as,

$$\frac{\partial f}{\partial A}=-\alpha S \text{ and } \frac{\partial f}{\partial S}=-\alpha A$$

$$\frac{\partial g}{\partial A}=\alpha S \text{ and } \frac{\partial g}{\partial S}=\alpha A-\beta$$

$$J(A,S)=\begin{pmatrix}-\alpha S & -\alpha A\\ \alpha S & \alpha S-\beta\end{pmatrix}$$

$$J\left(\frac{\beta}{\alpha},\frac{\mu}{\beta}\right)=\begin{pmatrix}\frac{-\alpha\mu}{\beta} & -\beta\\ \frac{\alpha\mu}{\beta} & 0\end{pmatrix}$$

$$|\,A-\lambda I\,|=0$$

$$\left|\begin{pmatrix}\frac{-\alpha\mu}{\beta} & -\beta\\ \frac{\alpha\mu}{\beta} & 0\end{pmatrix}-\lambda\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right|=0$$

$$\begin{vmatrix}\frac{-\alpha\mu}{\beta}-\lambda & -\beta\\ \frac{\alpha\mu}{\beta} & -\lambda\end{vmatrix}=0$$

$$-\lambda\left(\frac{-\alpha\mu}{\beta}-\lambda\right)+\alpha\mu=0$$

$$\lambda^2+\left(\frac{\alpha\mu}{\beta}\right)\lambda+\alpha\mu=0$$

From the general Quadratic formula,

$$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Where $a=1, b=\frac{\alpha\mu}{\beta}, c=\alpha\mu$

$$\lambda = \frac{-\frac{\alpha\mu}{\beta} \pm \sqrt{\left(\frac{\alpha\mu}{\beta}\right)^2 - 4\alpha\mu}}{2} \qquad (18)$$

By looking at the eigenvalues of this matrix, we can determine the stability of the original nonlinear system at the equilibrium. This implies the original nonlinear system would behave like the linearized system at the equilibrium. The eigenvalues of the *Jacobian*, $\lambda$ calculated as,

$$\lambda = \frac{-\frac{\alpha\mu}{\beta} \pm \sqrt{\left(\frac{\alpha\mu}{\beta}\right)^2 - 4\alpha\mu}}{2}$$

Since the eigenvalues, $\lambda$ are determined by the parameters, we need to break down into two cases to determine the stability of the equilibrium.

First, When $\frac{\alpha\mu}{\beta^2} \geq 4$ (19)

The eigenvalues $\lambda$ have two negative real parts since we know,

$$\frac{\alpha\mu}{\beta} > \sqrt{\left(\frac{\alpha\mu}{\beta}\right)^2 - 4\alpha\mu} > 0.$$

Hence the system forms a nodal sink at the equilibrium. That implies the system have the behavior of exponential decay at the equilibrium, $E(A^*, S^*) = \left({}^{\beta}/_{\alpha}, {}^{\mu}/_{\beta}\right)$ which is asymptotically stable. Thus, every solution of the system will approach to the stable equilibrium point, $A^* = {}^{\beta}/_{\alpha},$ and $S^* = {}^{\mu}/_{\beta}$, as time $t$ goes to infinity.

Secondly, When $\frac{\alpha\mu}{\beta^2} < 4$ (20)

The eigenvalues, $\lambda$, have complex parts along with a negative real part, hence the system forms a spiral sink at the equilibrium. That means the system have a certain form of oscillation behavior at equilibrium, $E(A^*, S^*) = \left({}^{\beta}/_{\alpha}, {}^{\mu}/_{\beta}\right)$, while its solutions still move to stable direction. In other words, the system behaves in a damped oscillatory manner with a certain period determined by the parameters. Given the model parameters, the period of the oscillation plays a role for us to predict the further behavior of the infection.

Interestingly, in either case, we can observe a stable equilibrium point, $A^* = {}^{\beta}/_{\alpha}$, and $S^* = {}^{\mu}/_{\beta}$, where the violence would have a steady state. Now, an interesting question comes out. The parametric conditions with inequality, (19) and (20), come purely from the mathematical analysis and we have no clear insight about how they are related to the behaviour of the violence spread. Keeping this in mind, we will reconsider the system of the differential equations, (13) and (14), which originally gives us a general picture of the violence interaction. If we non-dimensionalized the system,

$$\frac{dA}{dt} = \mu - \alpha AS$$

$$\frac{dS}{dt} = \alpha AS - \beta S$$

With the characteristic scales $c_A = \frac{\beta}{\alpha}$ for the Abusive class A, $c_S = \frac{\beta}{\alpha}$ for the Susceptible and Domestic Violence Victim class S, and $c_T = \frac{1}{\beta}$ for Time t, we end up with a non-dimensionalized system of the differential equations as the following.

$$\frac{d\tilde{A}}{d\tilde{t}} = \frac{\alpha\mu}{\beta^2} - \tilde{A}\hat{S} \qquad (21)$$

$$\frac{d\hat{S}}{d\tilde{t}} = \hat{S}(\tilde{A} - 1) \qquad (22)$$

Where $\frac{\alpha\mu}{\beta^2}$ is a free parameter, $\rho$.

Now, we can easily notice that the free parameter, $\rho$, in the non-dimensionalized system is same as the parametric conditions, (19) and (20). That means the overall behavior of the violence can be observed by changing the free parameter, $\rho = \frac{\alpha\mu}{\beta^2}$ with the non-dimensionalized equations. If we simply look at the non-dimensionalized violence epidemic model, the free parameter, $\rho$, can be interpreted as recruitment to the population of a Violence-high-risk Tamale metropolitan, $N$; of course, the recruitment then immediately belongs to the Abusive class, $A$, as the violence spread is in progress. This phenomenon is similar to the fact that the original differential equations, (13) and (14), has the $\mu$ term, which represents new entrance to the Abusive class per unit time. Note that, in the dimensionless system of differential equations, the dimension of $\mu$, $[\mu] = \frac{P}{T}$, where $[A] = [S] = P$ and $[t] = T$.

Recall the "*forcing term*" that we mentioned earlier. It becomes now clearer that the $\mu$ term acts like a "*forcing term*" which makes the behavior of the violence change. In fact, it is more accurate to say the free parameter, $\rho = \frac{\alpha\mu}{\beta^2}$, determines the behavior of the overall violence epidemic.

When $\rho \geq 4$, the solutions in the phase-plane move to the nodal sink equilibrium, $(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$.

When $\rho < 4$, the solutions in the phase-plane move to the spiral sink equilibrium, $E(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$.

The general behavior of the violence epidemic as shown in the figures is predicted under the initial condition of the violence epidemic, starting close enough to the equilibrium point, and as time goes to infinity. However, what happens the very beginning of the violence epidemic? The question might be a more realistic concern especially for the short history of Domestic Violence infection reflected on the real data in table (4.1). Consider the differential equation (14) of the system,

$$\frac{dS}{dt} = \alpha AS - \beta S$$

At the very beginning of the violence epidemic, the populations of the susceptible individuals$(S)$ and the Domestic Violence Victims$(V)$ are almost negligible compared to the Abusive individuals$(A)$. Therefore, $N = A + S + V$, becomes $N \approx A$. Besides, the relatively long incubation time of Domestic Violence to the insignificance of the term $-\beta S$ which represent the conversion from the Susceptible to Domestic Violence Victim. Ignorance of Domestic Violence with most cases not been reported could also cause $-\beta S \approx 0$ during that particular time span. Now, according to these assumptions, we have an equivalent differential equation,

$$\frac{dS}{dt} = \alpha NS \qquad (23)$$

By inspection, we can easily obtain the solution of the equation (21) is $S = S_0 e^{\alpha Nt}$. Clearly, $\alpha > 0$ as long as the epidemic is in effect and $N > 0$ since it's a population. Therefore, no matter how small the parameters are, the domestic Violence spread will exponentially grow in the beginning of the epidemic. This is what's actually happening in the real world at least in last 13 years according to the data from DOVVSU.

### 3.2 ANALYSIS OF THE LOGISTIC GROWTH MODEL

#### 3.2.1 Graph the Data

Considering the population sizes for domestic Violence victims in Tamale for the years between 1999 and 2011, we will derive a mathematical model for the Victims. Using the data below, we can plot the graph below.

Table 1: Table showing cases of domestic violence in Tamale from 1999 - 2011

| YEARS | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CASES | 15 | 27 | 53 | 94 | 128 | 142 | 158 | 161 | 172 | 174 | 181 | 187 | 191 |


Figure 6: Figure showing cases of domestic violence in Tamale from 1999 – 2011

#### 3.2.2 Logistic Model for the Data

Let's consider that the logistic growth model with form:

$$V(t)' = \beta V(t)\left(1 - \frac{V(t)}{N}\right) \qquad (24)$$

In order to show that model (13) is logistic, we need to focus on the following questions:

1) How to tell whether a given set of data is reasonably logistic?
2) What parameter $\beta$ and $N$ will be good fit?

Since we have discrete data, then we describe the model using a difference equation. We use previous values from the system to calculate the new ones. The equation (24) can be expressed by the difference equation version as the following equation:

$$V(t+1) - V(t) = \beta V\left(1 - \frac{V}{N}\right) \qquad (25)$$

It can be rewritten as:

$$\frac{\Delta V}{V} = \beta\left(1 - \frac{V}{N}\right) \qquad (26)$$

The equation (26) says that the ratio of $\Delta V$ and $V$ is a linear function of $V$.

Now we have testing of logistic behaviour for the model:

First of all, let's consider the left hand side (LHS) of equation (26). We calculate the difference of the populations for two consecutive years, and then use those differences against the corresponding function values. Next, we plot the ratios and the corresponding function values. At last, if we can show that the plots are approximately linear, then the model equation (26) is reasonable. That is to say, the model has the form (24) and it is Logistic.

Calculating the ratios on the left hand side of (24) yields:

$$a_1 = \frac{V(2000) - V(1999)}{V(1999)} = \frac{27-15}{15} = 0.8000;$$

Thus, we have the following list of data:

Table 2: Table showing the ratios and the corresponding function values

| $V(t)$ | 15 | 27 | 53 | 94 | 128 | 142 | 158 | 161 | 172 | 174 | 181 | 187 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a = \frac{\Delta V(t)}{V(t)}$ | 0.8000 | 0.9629 | 0.7736 | 0.3617 | 0.1094 | 0.1127 | 0.0189 | 0.0683 | 0.0116 | 0.0402 | 0.0331 | 0.0214 |

Plotting the Least Square approximation graph by using the data from Table (4.2) obtain following graph.

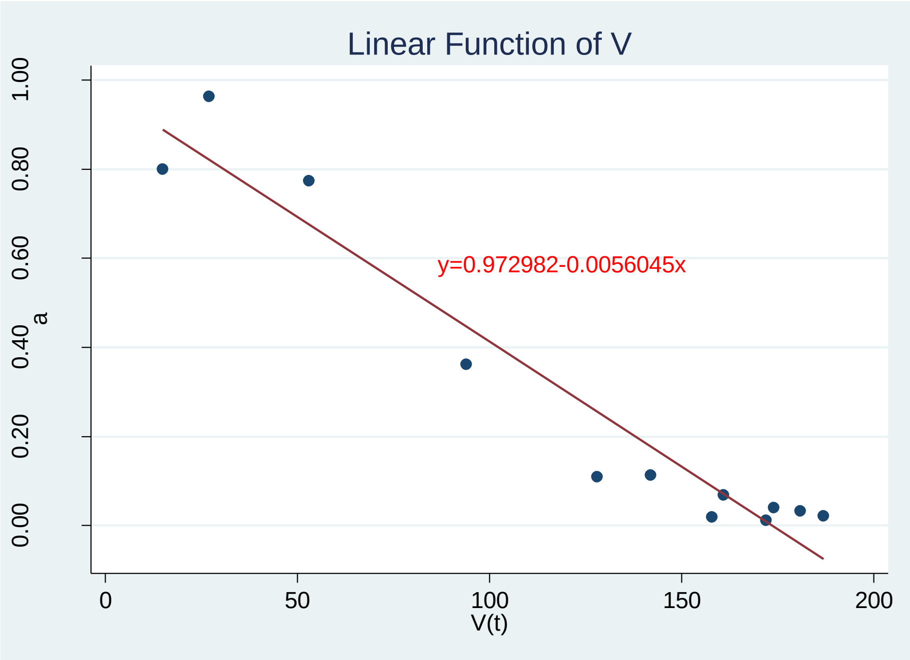


Figure 7: Figure showing linear function of V

As we can see in Figure (4.7), at various cases of Domestic Violence Victim plotted levels $V(t)$ at time $t$, we can calculate corresponding ratios $a$ . Based on these points we plot Least Square Approximation graph.

Looking at the graph, we can see that most of the data points are close to this line. The overall resulting plot is approximately linear. Therefore, our assumption for the equation (24) is reasonable. That is the present model (24) shows that the given data is logistic.

### 3.2.3 Determining the Values of $\beta$ and $N$

In the Least Square Approximation graph, figure (7), we know the equation for the line, which is,

$$y = 0.972982 - 0.0056045x$$

(27)

Substituting the point $V(1999)$ into this equation, we obtain,

$$y_1 = 0.972982 - 0.0056045(15)$$

$$y_1 = 0.888915$$

Similarly, substituting $V(2000)$ into it, we obtain

$$y_2 = 0.972982 - 0.0056045(27)$$

$$y_2 = 0.821661$$

That is to say, we can get values of the ratio, $a$, where $y = a = \frac{\Delta V}{V}$.

Substituting the data of 1999 and 2000 to the equation (4.14) gives,

$$0.888915 = \beta\left(1 - \frac{15}{N}\right) \qquad (28)$$

$$0.821661 = \beta\left(1 - \frac{27}{N}\right) \qquad (29)$$

Suppose that $\beta, N \neq 0$, and divide equation (28) by equation (29), we can get that:

$$\frac{0.888915}{0.821661} = \frac{\beta\left(1-\frac{15}{N}\right)}{\beta\left(1-\frac{27}{N}\right)}$$

$$N = 173.6103\ , \quad N \approx 174$$

From equation (28), we obtain the value of $\beta$,

$$0.888915 = \beta\left(1 - \frac{15}{174}\right)$$

$$\beta = 0.972775$$

Therefore, the model is

$$V(t)' = 0.972775V\left(1 - \frac{V}{174}\right) \qquad (30)$$

As we know, the size for the number of cases tends to the carrying capacity $N$. In this case, the size is bounded by 174 victims of Domestic Violence. In another words, the limiting number for this population model is 174 Victims.

### 3.2.4 The Solution for the Logistic Model

Rewriting equation (4.18), we have

$$\frac{dV}{dt} = 0.972775V\left(1 - \frac{V}{174}\right)$$

$$\frac{dV}{dt} = 0.972775V - 0.005591V^2 \qquad (31)$$

By separation of variables, equation (4.19) gives

$$\int \frac{dV}{V(0.972775 - 0.005591V)} = t + c \qquad (32)$$

Also because

$$\frac{1}{V(0.972775 - 0.005591V)} = \frac{1}{0.972775}\left(\frac{1}{V} + \frac{0.005591}{0.972775 - 0.005591V}\right)$$

The equation (4.20) can be written as

$$\frac{1}{0.972775}\int\left(\frac{1}{V} + \frac{0.005591}{0.972775 - 0.005591V}\right)dV = t + c$$

Let $t = 0$ corresponds to the size of cases in 1999, 15. Then we have $V_0 = 15$

Substituting the condition $V_0 = 15$ at $t = 0$ and integrating, we get

$$c = \frac{1}{0.972775}[\ln(15) - \ln(0.972775 - 0.005591(15))]$$

$$c = 2.90489$$

Thus equation (4.20) becomes

$$\frac{1}{0.972775}(\ln V - \ln(0.972775 - 0.005591V)) = t + 2.9$$

$$V = \frac{16.4595e^{0.972775t}}{1 + 0.0946e^{0.972775t}}$$

Dividing through by $0.0946e^{0.972775t}$ gives

$$V = \frac{173.9905}{1 + 10.5708e^{-0.972775t}} \qquad (33)$$

If we take the limit of solution (33) as $t \to \infty$, we can see that, $V(t) \to 174$, this shows that there is a limit to the growth of $V$. The limiting number is 174 Victims. From equation (33), we can get the predicted cases for each year. **TABLE 3** shows time$(t)$, actual cases$(V)$ and predicted cases$(V(t))$ for Domestic Violence against women and children.

Table 3: Table showing time$(t)$, actual cases$(V)$ and predicted cases$(V(t))$ for Domestic Violence against women and children.

| Time (t) | Actual Cases V | Predicted Cases V(t) |
|---|---|---|
| 0 | 15 | 15.0379 |
| 1 | 27 | 34.8271 |
| 2 | 53 | 69.3045 |
| 3 | 94 | 110.5040 |
| 4 | 128 | 143.1060 |
| 5 | 142 | 160.8710 |
| 6 | 158 | 168.7920 |
| 7 | 161 | 171.9940 |
| 8 | 172 | 173.2360 |

| | | |
|---|---|---|
| 9 | 174 | 173.7100 |
| 10 | 181 | 173.8900 |
| 11 | 187 | 173.9590 |
| 12 | 191 | 173.9840 |
| 13 | | 173.9940 |
| 14 | | 173.9980 |
| 15 | | 173.9990 |
| 16 | | 174 |
| 17 | | 174 |
| 18 | | 174 |

As $t \to \infty$, we can see that, $V(t) \to 174$.

Assuming now, the carrying capacity, $N = 1000$.

Then from equation (28) we obtain the value of $\beta$;

$$0.888915 = \beta\left(1 - \frac{15}{1000}\right)$$

$$\beta = 0.902452$$

Then, the model becomes

$$V(t)' = 0.902452V\left(1 - \frac{V}{1000}\right) \qquad (34)$$

When solve gives

$$V(t) = \frac{1000.4900}{1 + 65.6987\, e^{-0.902452t}} \qquad (35)$$

From equation (35), the predicted cases of Domestic Violence is shown in **TABLE 4**.

| Time (t) | Predicted Cases V(t) |
|---|---|
| 0 | 14.992 |
| 1 | 36.172 |
| 2 | 84.697 |
| 3 | 185.772 |
| 4 | 360.021 |
| 5 | 581.073 |
| 6 | 773.754 |
| 7 | 893.983 |
| 8 | 954.110 |
| 9 | 980.866 |
| 10 | 992.151 |
| 11 | 996.802 |
| 12 | 998.700 |
| 13 | 999.472 |
| 14 | 999.786 |
| 15 | 999.913 |
| 16 | 999.965 |
| 17 | 999.986 |
| 18 | 999.994 |
| 19 | 999.998 |
| 20 | 999.999 |
| 21 | 1000.000 |
| 22 | 1000.000 |

Table 4: as $t \to \infty$, we can see that, $V(t) \to 1000$.

Finally, assuming the carrying capacity is equal to the population of females in the Northern region according to the provisional results of the Population and Housing Census of Ghana, 2010. That is $N = 1257855$

Then from equation (4.16) we obtain the value of $\beta$;

$$0.888915 = \beta\left(1 - \frac{15}{1257855}\right)$$

$$\beta = 0.888926$$

Then, the model becomes

$$V(t)' = 0.888926V\left(1 - \frac{V}{1257855}\right)$$

(36)

When solve gives

$$V(t) = \frac{1250008.3333}{1 + 83333.3333\, e^{-0.888926t}} \qquad (37)$$

From equation (37), we obtain the predicted cases of Domestic Violence in the **TABLE 5** below;

| Time (t) | Predicted Cases V(t) |
|---|---|
| 0 | 14.9999 |
| 1 | 36.4860 |

| | |
|---|---|
| 2 | 88.7512 |
| 3 | 215.8669 |
| 4 | 524.9698 |
| 5 | 1276.2295 |
| 6 | 3099.9146 |
| 7 | 7513.8979 |
| 8 | 18121.6303 |
| 9 | 43184.3213 |
| 10 | 100092.9939 |
| 11 | 218423.1079 |
| 12 | 424947.3826 |
| 13 | 695155.9456 |
| 14 | 941182.6164 |
| 15 | 1101434.4142 |
| 16 | 1184333.0296 |
| 17 | 1222147.3674 |
| 18 | 1238402.4356 |
| 19 | 1245210.9520 |
| 20 | 1248031.6770 |
| 21 | 1249194.9782 |
| 22 | 1249673.8372 |
| 23 | 1249870.8013 |
| 24 | 1249951.7906 |
| 25 | 1249995.0881 |
| 26 | 1249998.7771 |
| 27 | 1250004.4047 |
| 28 | 1250006.7183 |
| 29 | 1250007.6693 |
| 30 | 1250008.0603 |
| 31 | 1250008.2211 |
| 32 | 1250008.2871 |
| 33 | 1250008.3143 |
| 34 | 1250008.3255 |
| 35 | 1250008.3301 |
| 36 | 1250008.3319 |
| 37 | 1250008.3327 |
| 38 | 1250008.3330 |
| 39 | 1250008.3332 |
| 40 | 1250008.333 |

Table 5: as $t \to \infty$, we can see that, $V(t) \to 1250008$.

In conclusion, equation (33), (35) and (37) shows that there is a limit to the growth Domestic Violence Cases in Tamale Metropolitan.

## 4.0 CONCLUSION

A simple continuous model can mirror a primary interaction of Domestic Violence spread, using a system of Ordinary Differential Equations called the Violence Epidemic Model (ASV). The modeling is accomplished by formulating the rate of change of each population with their interactions. According to the results, by changing an environmental-control parameter, $\rho$, the long-term behavior of Domestic Violence against Women and children spread changes until the spread reaches a steady state. When $\rho \geq 4$, the solutions in the phase-plane move to the nodal sink equilibrium state $E(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$.

When $\rho < 4$, the solutions in the phase-plane move to the spiral sink equilibrium state $E(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$. The result also shows that the Violence spreads in the very beginning obeying the Exponential growth Model $S = S_0 e^{\alpha N t}$.

Knowing the simplified model might be somewhat inappropriate for the dynamics of Domestic Violence spread, though, fundamental mathematical modeling techniques are accomplished in this research to conceptualize a physical phenomenon. Analytic approaches are conveniently used to understand the factors governing the behavior of the model.

Also, the values collected from Domestic Violence and Victim Support Unit of the Ghana Police, Tamale was tested using the Logistic Growth Model;

$V(t)' = 0.972775V\left(1 - \frac{V}{174}\right)$ And the solution is $V = \frac{173.9905}{1+10.5708e^{-0.972775t}}$. The solution shows that there is a limit to the growth of Domestic Violence Victims $V$ as time $t \to \infty$. The limiting number is 174 Victims. Assumptions of different carrying capacity of Tamale Metropolitan were considered and its Logistic Growth Model was deduced. Their result shows that there is a limit to the growth of Domestic Violence Victims over time period. The Logistic Growth Model was tested using data values and it proved to predict accurate figures of Domestic Violence Victims.

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